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Boundary CFT construction of D-branes in vacuum string field theory

Leonardo Rastelli

Department of Physics Princeton University
Princeton, NJ 08540, USA
E-mail: rastelli@feynman.princeton.edu

Ashoke Sen

Harish-Chandra Research Institute
Chhatnag Road, Jhusi, Allahabad 211019, India, and
Institute for Theoretical Physics
University of California, Santa Barbara, CA 93106, USA
E-mail: asen@thwgs.cern.ch, sen@mri.ernet.in

Barton Zwiebach

Center for Theoretical Physics
Massachussetts Institute of Technology
Cambridge, MA 02139, USA
E-mail: zwiebach@mitlns.mit.edu

Abstract: In previous papers we built (multiple) D-branes in flat space-time as classical solutions of the string field theory based on the tachyon vacuum. In this paper we construct classical solutions describing all D-branes in any fixed space-time background defined by a two dimensional CFT of central charge 26. A key role is played by the geometrical definition of the sliver state in general boundary CFT's. The correct values for ratios of D-brane tensions arise because the norm of the sliver solution is naturally related to the disk partition function of the appropriate boundary CFT. We also explore the possibility of reproducing the known spectrum of physical states on a D-brane as deformations of the sliver.

Keywords: Bosonic Strings, D-branes.

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1. Introduction and summary

Much work has been done in understanding various conjectures about tachyon condensation on D-branes in bosonic string theory [1, 2, 3] using cubic open string field theory [4] (SFT). Although the results are very impressive, they ultimately rely on numerical study of the solutions of the equations of motion using the level truncation scheme [5]–[24].[1] In a series of three papers [29]–[31] we attempted an analytic

[1]For some early attempts at understanding the open string tachyon, see refs. [25]. For field theory models of tachyon condensation, see refs. [26, 27]. Study of tachyon condensation using renormalization group approach has been carried out in refs. [28].

approach to the issues of tachyon condensation by proposing a candidate string field theory action which describes string field theory expanded around the tachyon vacuum. As opposed to the conventional cubic SFT where the kinetic operator is the BRST operator Q_B, here the kinetic operator $\mathcal{Q}$ is non-dynamical and is built solely out of worldsheet ghost fields.[2] In this class of actions the absence of physical open string states around the vacuum is manifest. Gauge invariance holds, and therefore basic consistency requirements are expected to be satisfied. Furthermore we showed that this theory contains classical solutions representing Dp-branes for all $p \leq 25$, with correct ratios of tensions, thereby providing a non-trivial check on the correctness of our proposal. The key ansatz that made this analysis possible was that the string field solution representing a D-brane factorizes into a ghost part Ψ_g and a matter part Ψ_m, with Ψ_g the same for all D-branes, and Ψ_m different for different D-branes. The Ψ_m representing the D25-brane is a particular surface state of the corresponding boundary conformal field theory (BCFT) known as the sliver [11, 33, 30, 31].

In this paper we discuss the construction of D-brane solutions in this theory using conformal field theory techniques. This construction has certain conceptual and practical advantages. In the construction of ref. [30] we obtained an expression for the ratio of tensions of different D-branes in terms of ratios of determinants of infinite matrices. There was no analytic understanding, however, why this ratio gave the expected answer. In the approach taken here, the correct values of all ratios of tensions are obtained manifestly. Furthermore the analysis given in this paper can be carried out in the background of any bulk conformal field theory (CFT), for any D-brane described by an appropriate BCFT.[3]

We start with some fixed space-time background described by a particular bulk CFT, and choose once and for all a specific reference BCFT, denoted as BCFT_0. We define our string field to be a state in the Hilbert space of BCFT_0. In the analysis of refs. [29, 30, 31] the D25-brane BCFT played the role of BCFT_0. As in those works, we seek solutions of the form $\Psi_g \otimes \Psi_m$, where the ghost component Ψ_g is universal, but the matter part Ψ_m varies from one solution to another. Under this factorization hypothesis, the matter part of the string field satisfies a simple equation: it squares to itself under $*$-multiplication. One particular solution of this equation is the matter part of the sliver state of BCFT_0. We identify this state as the solution representing the D-brane associated with BCFT_0, generalizing the identification of the D25-brane solution as the sliver of the corresponding BCFT [30, 31].

[2] A subset of this class of actions was discussed previously in ref. [32].

[3] Throughout this paper a specific BCFT will refer to the boundary conformal field theory associated with a *single* D-brane. Multiple D-brane solutions will be obtained by first constructing solutions corresponding to the individual D-branes, and then taking appropriate superposition of the solutions.

In the conformal field theory description [11], the sliver is regarded as a surface state by using the standard procedure for associating a BCFT state to every Riemann surface with a boundary, with one puncture at the boundary and a local coordinate at the puncture. The sliver is described as the surface state associated with a specific once-punctured disk. This description of the sliver is universal in the sense that the state takes exactly the same form for any BCFT when written in terms of the Virasoro operators of the BCFT. It is therefore natural to expect that, just as in the case of BCFT_0, the sliver of an arbitrary BCFT describes the D-brane associated with that BCFT. In order to regard different D-branes as different solutions in the SFT, however, we need to express the slivers associated with these different BCFT's as states in the Hilbert space of BCFT_0. We find an explicit algorithm for doing this. The solution constructed this way correctly reproduces the tension of the D-brane associated with the specific BCFT (up to an overall normalization constant which is the same for all D-branes). This is a nontrivial result that follows because the tension, in vacuum SFT, is given by the BPZ inner product of the solution with itself, and we show that this inner product is simply related to the disk partition function in the specific BCFT. The identification of the disk partition function with the D-brane tension is a well-known result [34]–[38] that has played a crucial role in the study of tachyon condensation in boundary string field theory [39]–[48].

The paper is organized as follows. In section 2 we give a brief summary of the results of refs. [29, 30]. In section 3 we give a detailed review of the construction of the sliver as a surface state associated with a once-punctured disk with a local coordinate at the puncture. We describe it in various coordinate systems which are useful in our analysis. We also explain why it is a well defined state, in that the inner product of the sliver with any BCFT state associated with a vertex operator is computable and finite. This is not a priori obvious, since the state arises in a limit where the local coordinate at the puncture becomes singular. Nevertheless, $SL(2,\mathbb{R})$ invariances of the state allow geometrical representations where the local coordinate is non-singular, and the resulting state is manifestly regular. It is significant that even the regular geometrical description of the sliver is unusual in that the open string midpoint, which usually is somewhere inside the disk, reaches the boundary of the disk. This fact gives an intuitive understanding of the left-right factorization of the sliver functional, a key result in the analysis of [31, 49]. Our discussion also addresses star multiplication of surface states, and explains why the sliver squares to itself under star multiplication.

In section 4 we use the universal description of the sliver to construct solutions of the SFT equations of motion describing different D-branes in some fixed space-time background. We take for the space-time background some fixed bulk CFT with central charge 26, and choose a reference boundary conformal field theory BCFT_0, in whose state space the string field takes value. The matter part of the sliver of BCFT_0 is the matter part of a solution describing the D-brane corresponding

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to BCFT_0. We show in section 4.1 that every other D-brane in this space-time background, described by some other boundary conformal field theory BCFT', can also be obtained as a solution in this SFT. We explicitly write down the classical solution describing this D-brane, and verify that it satisfies the equations of motion and correctly reproduces the tension of the D-brane. In section 4.2 we show that solutions describing D-branes associated with two *different* BCFT's $*$-multiply to zero, and hence we can construct multiple D-brane solutions by superposing them. We also give a construction of identical coincident D-branes.

In section 4.3 we discuss the construction of classical solutions corresponding to a two dimensional field theory obtained by deforming BCFT_0 by a relevant or marginal boundary operator. The results bear strong resemblance to those of boundary string field theory, except for one important difference. In boundary string field theory, the coefficient of a relevant deformation is driven to infinity (or more generally to the infrared fixed point) by the equations of motion. In contrast, here we get a solution of the equations of motion for arbitrary value of this coefficient. When we compute the tension of the corresponding solution in section 4.4, however, we recover the partition function on a disk of the deformed boundary *conformal* field theory (which we call BCFT'), with the coefficient of the perturbation driven to its infrared fixed point due to a conformal transformation involving infinite rescaling. Thus the conclusion is that different values of these coefficients describe the same D-brane solution − the one associated with BCFT'.[4] For exactly marginal deformations, of course, different values of the coefficient represent genuinely different solutions.

If BCFT' and BCFT_0 are related by an exactly marginal deformation, we can also study small deformations of the solution by taking the perturbing parameter to be small. This is done in section 4.5. We show in section 4.6 that these small deformations can be thought of as covariant derivatives of the sliver state with respect to a canonical theory-space connection introduced in ref. [51]. We also discuss the background independence of vacuum string field theory in the language of connections over theory space.

In section 5 we address the question of the spectrum of physical states around the solution describing the D-brane associated with BCFT_0. Since general excitations around this solution involve both matter and ghost oscillators, this problem cannot be studied completely without knowing the form of the kinetic operator $\mathcal{Q}$ and the ghost part Ψ_g of the D-brane solution. We make an attempt to study this problem in section 5.1 by restricting ourselves to excitations of the factorized type with the same universal ghost part Ψ_g but arbitrary fluctuations in the matter sector. We find that we do get solutions of the linearized equations of motion for every dimension-one primary in the matter part of BCFT_0, as is expected of the open string spectrum on a D-brane. We also need to determine which of these solutions could be re-

[4]This had been earlier anticipated by Witten [50].

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lated by linearized gauge transformations. Again, without detailed knowledge of $\mathcal{Q}$, we proceed with a restricted class of gauge transformation parameters described in section 5.2 which preserve the factorized form of the linearized fluctuations. Section 5.3 describes some subtleties that arise in studying the normalization of states. We discuss a plausible resolution leading to some interesting conclusions. First, it is strongly suggested that the kinetic operator $\mathcal{Q}$ of vacuum SFT must annihilate the identity operator. Second, we find that any linearized solution to the equations of motion with finite BPZ norm is pure gauge. Physical states are then argued to be associated with states whose BPZ norm diverges logarithmically in the parameters defining finite versions of the sliver. Although these criteria lead us to identify sliver deformations by dimension-one primaries as physical states, it is not clear how dimension one nulls and non-primary operators are removed from the spectrum.

We conclude in section 6 with a discussion of some general aspects of the string field theory around the tachyon vacuum.

2. Review of vacuum string field theory

In this section we shall briefly describe the results of refs. [29, 30]. In these papers we proposed a form of the string field theory action around the open bosonic string tachyon vacuum and discussed classical solutions describing D-branes of various dimensions. In order to write concretely this theory we choose to use the state space $\mathcal{H}$ of the combined matter-ghost boundary conformal field theory (BCFT) describing the D25-brane. The string field Ψ is a state of ghost number one in $\mathcal{H}$ and the string field action is given by:

$$\mathcal{S}(\Psi) \equiv -\frac{1}{g_0^2}\left[\frac{1}{2}\langle\Psi, \mathcal{Q}\,\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right], \tag{2.1}$$

where g_0 is the open string coupling constant, $\mathcal{Q}$ is an operator made purely of ghost fields, $\langle\, ,\, \rangle$ denotes the BPZ inner product, and $*$ denotes the usual $*$-product of the string fields [4]. $\mathcal{Q}$ satisfies the requirements:

$$\begin{aligned} \mathcal{Q}^2 &= 0\,, \\ \mathcal{Q}(A * B) &= (\mathcal{Q}A) * B + (-1)^A A * (\mathcal{Q}B)\,, \\ \langle \mathcal{Q}A, B\rangle &= -(-)^A \langle A, \mathcal{Q}B\rangle\,. \end{aligned} \tag{2.2}$$

The action (2.1) is then invariant under the gauge transformation:

$$\delta\Psi = \mathcal{Q}\Lambda + \Psi * \Lambda - \Lambda * \Psi\,, \tag{2.3}$$

for any ghost number zero state Λ in $\mathcal{H}$. Ref. [29] contains candidate operators $\mathcal{Q}$ satisfying these constraints; for our analysis we shall not need to make a specific choice of $\mathcal{Q}$. The equations of motion are

$$\mathcal{Q}\Psi + \Psi * \Psi = 0\,. \tag{2.4}$$

In ref. [30] we made the ansatz that all Dp-brane solutions in this theory have the factorized form:

$$\Psi = \Psi_g \otimes \Psi_m \,, \tag{2.5}$$

where Ψ_g denotes a state obtained by acting with the ghost oscillators on the $\mathrm{SL}(2,\mathbb{R})$ invariant vacuum of the ghost BCFT, and Ψ_m is a state obtained by acting with matter operators on the $\mathrm{SL}(2,\mathbb{R})$ invariant vacuum of the matter BCFT. Let us denote by $*^g$ and $*^m$ the star product in the ghost and matter sector respectively. Eq. (2.4) then factorizes as

$$\mathcal{Q}\Psi_g = -\Psi_g *^g \Psi_g \,, \tag{2.6}$$

and

$$\Psi_m = \Psi_m *^m \Psi_m \,. \tag{2.7}$$

We further assumed that the ghost part Ψ_g is universal for all Dp-brane solutions. Under this assumption the ratio of energies associated with two different D-brane solutions, with matter parts Ψ'_m and Ψ_m respectively, is given by:

$$\frac{\langle \Psi'_m | \Psi'_m \rangle_m}{\langle \Psi_m | \Psi_m \rangle_m} \,, \tag{2.8}$$

with $\langle \cdot | \cdot \rangle_m$ denoting BPZ inner product in the matter BCFT. Thus the ghost part drops out of this calculation.

In ref. [30] we constructed analytically the matter part of the solution for different Dp-branes, and verified numerically that we get the correct ratio of tensions of Dp-branes using eq. (2.8). The matter part of the D25-brane solution was given by the sliver state $|\Xi\rangle$ which will play an important role in the analysis of this paper. The construction of $|\Xi\rangle$ will be reviewed in section 3.

In this paper, we shall be using a more general setup. Instead of restricting ourselves to D-branes in flat space-time, we shall consider a general space-time background described by some arbitrary bulk CFT. The role of the D25-brane, in whose Hilbert space the string field takes value, is played by some fixed D-brane in this background associated with a specific BCFT. We shall call this BCFT_0. The matter part of $|\Xi\rangle$, described as a surface state in BCFT_0, then describes the D-brane associated with BCFT_0.

3. The various pictures of the sliver

In this section we will examine the sliver state $|\Xi\rangle$ from its geometrical definition. In this context we take the opportunity to describe in detail the various ways the sliver can be presented. We emphasize how, despite its origin as a surface state with a singular coordinate, it is a well-defined state satisfying $\Xi * \Xi = \Xi$. Although we are interested in the matter part of the sliver, we shall work in most part with the full sliver including the ghost sector, so that there is no central charge contribution to the various conformal transformations and gluing operations. Having established that $\Xi * \Xi = \Xi$, we can then use the factorization property of the $*$-product to conclude that $\Xi_m *^m \Xi_m = \Xi_m$ with suitable normalization of Ξ_m (which could be infinite, but is universal in the sense that it does not depend on the specific choice of matter BCFT).

3.1 Viewpoints on surface states

The sliver is a ghost number zero state that has a universal definition. It is a *surface state*, which means that for any given BCFT it can be defined as the bra $\langle\Xi|$ associated to a particular Riemann surface Σ. The surface in question is a disk D with one puncture P at the boundary. Moreover, there is a local coordinate at this puncture. The local coordinate at the puncture for the case of the sliver is obtained by a limiting procedure, to be reviewed and elaborated below.

We shall begin with a general discussion of surface states associated with a disk with one puncture. A local coordinate at a puncture is obtained from an analytic map m taking a canonical half-disk H_U defined as

$$H_U : \{|\xi| \leq 1, \mathrm{Im}(\xi) \geq 0\}\,, \tag{3.1}$$

into D, where $\xi = 0$ maps to the puncture P, and the image of the real segment $\{|\xi| \leq 1, \mathrm{Im}(\xi) = 0\}$ lies on the boundary of D. The coordinate ξ of the half disk is called the *local coordinate*. For any point $Q \in D$ in the image of the map, $\xi(m^{-1}(Q))$ is the local coordinate of the point. Using any *global coordinate* u on the disk D, the map m can be described by some analytic function s:

$$u = s(\xi)\,, \qquad u(P) = s(0)\,. \tag{3.2}$$

Figure 1 shows a disk D with a local coordinate around the puncture P. The image under m of the semicircle $|\xi| = 1$, corresponding to the curve AMB in figure 1 is

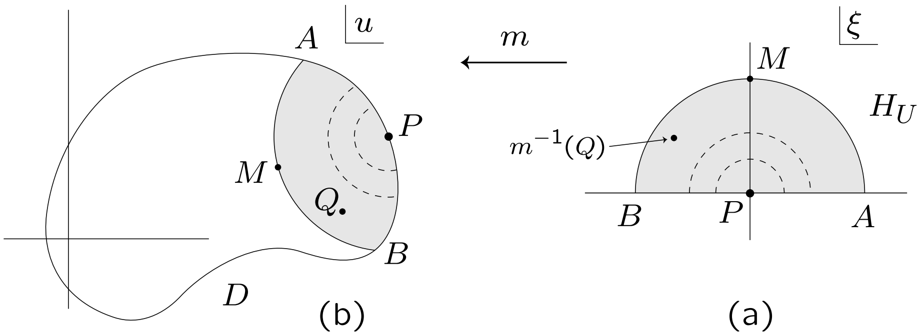

Figure 1: A punctured disk D with a local coordinate around the puncture P. The coordinate is defined through a map m from a canonical half disk H_U to the disk. The arcs AM and MB in D represent the left half and the right half of the open string respectively.

usually referred to as the open string. The image of point M, corresponding to $\xi = i$, is called the string midpoint. The image of the arc AM is called the left-half (as seen from the interior of D) of the open string, and similarly the image of MB is called the right-half of the open string. If we denote by $\sigma = \frac{1}{i}\ln\xi$ the coordinate along the string, then the left-half of the string corresponds to the region $0 \le \sigma < \pi/2$ and the right-half of the string corresponds to the region $\pi/2 < \sigma \le \pi$. The image of the half-disk H_U in the u-plane, shown by the shaded region in figure 1, will be called the *local coordinate patch.*

Given this geometrical data, and a BCFT with state space $\mathcal{H}$, the state $\langle\Sigma| \in \mathcal{H}^*$ associated to the surface Σ is defined as follows. For any local operator $\phi(\xi)$, with associated state $|\phi\rangle = \lim_{\xi\to 0}\phi(\xi)|0\rangle$ we set

$$\langle\Sigma|\phi\rangle = \langle s \circ \phi(0)\rangle_D\,, \tag{3.3}$$

where $\langle\,\rangle_D$ corresponds to correlation function on D and $s\circ\phi(0)$ denotes the conformal transform of the operator by the map $s(\xi)$, i.e. the operator $\phi(\xi = 0)$ expressed using the appropriate conformal map in terms of $\phi(s(0))$. For a primary of dimension h, $s \circ \phi(0) = \phi(s(0))(s'(0))^h$. The right hand side of eq. (3.3) can be interpreted as the one point function on D of the local operator ϕ inserted at P using the local coordinate ξ defined there. We also call, with a small abuse of notation, $|\Sigma\rangle \in \mathcal{H}$ a surface state; this is simply the BPZ conjugate of $\langle\Sigma|$. While computations of correlation functions involving states in $\mathcal{H}$ requires that the map s be defined only locally around the puncture P, more general constructions, such as the gluing of surfaces, an essential tool in the operator formulation of CFT, requires that the full map of the half disk H_U into the disk D be well defined.

At an intuitive level $\langle\Sigma|$ can be given the following functional integral representation. Consider the path integral over the basic elementary fields of the two dimensional conformal field theory, — collectively denoted as φ, — on the disk D minus the local coordinate patch, with some fixed boundary condition $\varphi = \varphi_0(\sigma)$ on the boundary AMB of the local coordinate patch, and the open string boundary condition corresponding to the BCFT under study on the rest of the boundary of this region. The parameter σ is the coordinate labeling the open string along AMB, defined through $\xi = e^{i\sigma}$. The result of this path integral will a functional of the boundary value $\varphi_0(\sigma)$. We identify this as the wave-functional of the state $\langle\Sigma|$. (For describing the wave-functional of $|\Sigma\rangle$ we need to make a $\sigma \to (\pi - \sigma)$ transformation.) On the other hand the wave-functional of the state $|\phi\rangle$ can be obtained by performing the path integral over φ on the unit half-disk in the ξ coordinate system, with the boundary condition $\varphi = \varphi_0(\sigma)$ on the semicircle, open string boundary condition corresponding to the BCFT on real axis, and a vertex operator $\phi(0)$ inserted at the origin. We can now compute $\langle\Sigma|\phi\rangle$ for any state $|\phi\rangle$ in $\mathcal{H}$ by multiplying the two wave-functionals and integrating over the argument $\varphi_0(\sigma)$. The net result is a path integration over φ on the full disk D, with the

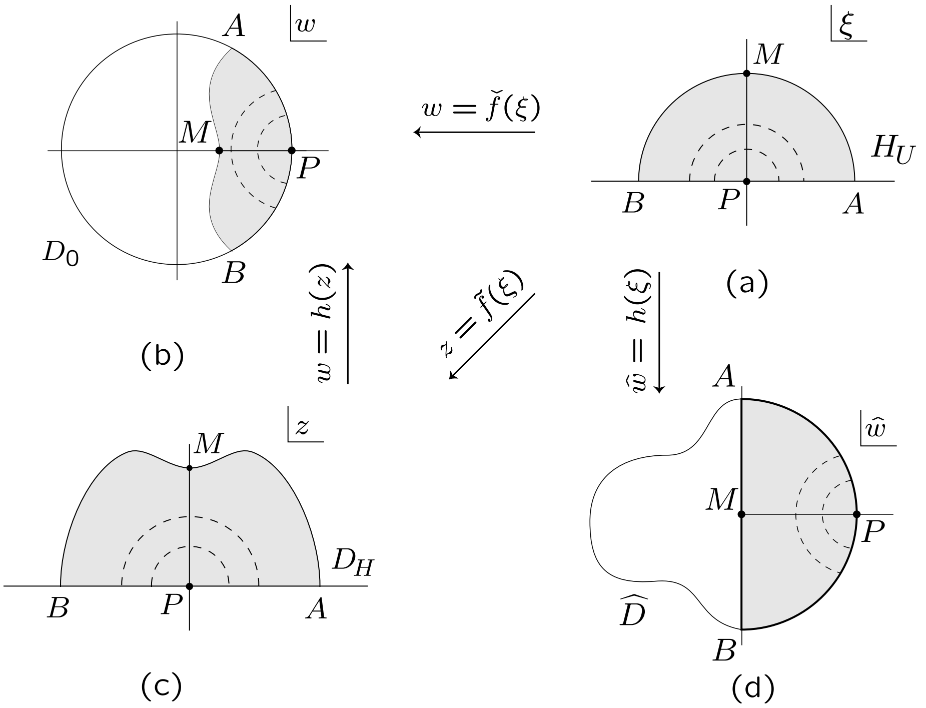

Figure 2: Three canonical presentations of the disk D. (b) Disk presented as the unit disk D_0 with global coordinate w. (c) Disk presented as the upper half plane D_H with global coordinate z. (d) Disk presented as $\widehat{D}$ with global coordinate $\widehat{w}$. Here the image of H_U is also a half-disk.

boundary condition corresponding to BCFT over the full boundary and a vertex operator ϕ inserted at the puncture P *using the ξ coordinate system.* This is precisely eq. (3.3).

For future use, we shall now describe three canonical ways of defining the surface state $\langle\Sigma|$, using three presentations of the disk D. In the first one, we present D as the unit disk $D_0 : |w| \leq 1$ in a w-plane. The puncture will be located at $w = 1$, and the local coordinate described as

$$w = \check{f}(\xi)\,, \qquad w(P) = \check{f}(0) = 1\,. \tag{3.4}$$

This is shown in figure 2b. In this presentation we have that eq. (3.3) takes the form

$$\langle\Sigma|\phi\rangle = \langle \check{f} \circ \phi(0)\rangle_{D_0}\,. \tag{3.5}$$

In the second presentation of D we map it to the upper half plane (UHP) with global coordinate z, and locate the puncture at $z = 0$. We will denote this upper half-plane as D_H. More concretely, we define z through the relation

$$w = h(z) \equiv \frac{1 + iz}{1 - iz}\,, \tag{3.6}$$

which maps the UHP, labeled by z, to the unit disk D_0. We then have that the local coordinate around P in this presentation takes the form:

$$z = h^{-1}(w) = h^{-1}(\check{f}(\xi)) \equiv \widetilde{f}(\xi)\,. \tag{3.7}$$

In this presentation eq. (3.3) can be rewritten as

$$\langle\Sigma|\phi\rangle = \langle \widetilde{f} \circ \phi(0)\rangle_{D_H}\,, \tag{3.8}$$

where $\langle\ \rangle_{D_H}$ denotes correlation function on the upper half plane. The disk D_H with its coordinate is shown in figure 2*c*.

Finally, we introduce the third presentation, where the disk D is mapped into a disk $\widehat{D}$ having the special property that the local coordinate patch, i.e. the image of the half disk H_U in $\widehat{D}$, is particularly simple. With global coordinate $\widehat{w}$, the image of H_U appears as a vertical half-disk of unit radius, with the curved part of H_U mapped to the imaginary axis and the diameter of H_U mapped to the unit semi-circle to the right of the imaginary axis (see figure 2*d*). This is achieved by taking, for $\xi \in H_U$,

$$\widehat{w} = h(\xi) = \frac{1+i\xi}{1-i\xi}\,. \tag{3.9}$$

In this presentation the rest of $\widehat{D}$ may take a complicated form. We can now rewrite eq. (3.3) as

$$\langle\Sigma|\phi\rangle = \langle h \circ \phi(0)\rangle_{\widehat{D}}\,, \tag{3.10}$$

where $\langle\ \rangle_{\widehat{D}}$ denotes the correlation function on the disk $\widehat{D}$ with appropriate open string boundary condition at the boundary of $\widehat{D}$. In this description the information about the state is encoded in the shape of the disk $\widehat{D}$.

3.2 The sliver surface state defined

We can now define the sliver state following the route originally taken through a limit of certain 'wedge states' [11]. We shall give this description in all three pictures by explicitly specifying the maps $\check{f}(\xi)$, $\widetilde{f}(\xi)$ and the disk $\widehat{D}$. In doing so we will refer to figures 3, 4 and 5. We begin by giving the description on the unit disk D_0. We define for any positive real number $n > 0$

$$w_n = \check{f}_n(\xi) \equiv (h(\xi))^{2/n} = \left(\frac{1+i\xi}{1-i\xi}\right)^{2/n}, \tag{3.11}$$

which for later purposes we also write as

$$w_n = \exp\left(i\,\frac{4}{n}\,\tan^{-1}(\xi)\right). \tag{3.12}$$

As already pointed out before, the map $h(\xi)$ takes the canonical half disk into a similar presentation, with the puncture now on the curved side of a half-disk (figure 3*b*). Moreover the string midpoint M at $\xi = i$ is mapped to $h(i) = 0$. The map $w_n = (h(\xi))^{2/n}$ makes the image of the canonical half-disk into a wedge with the

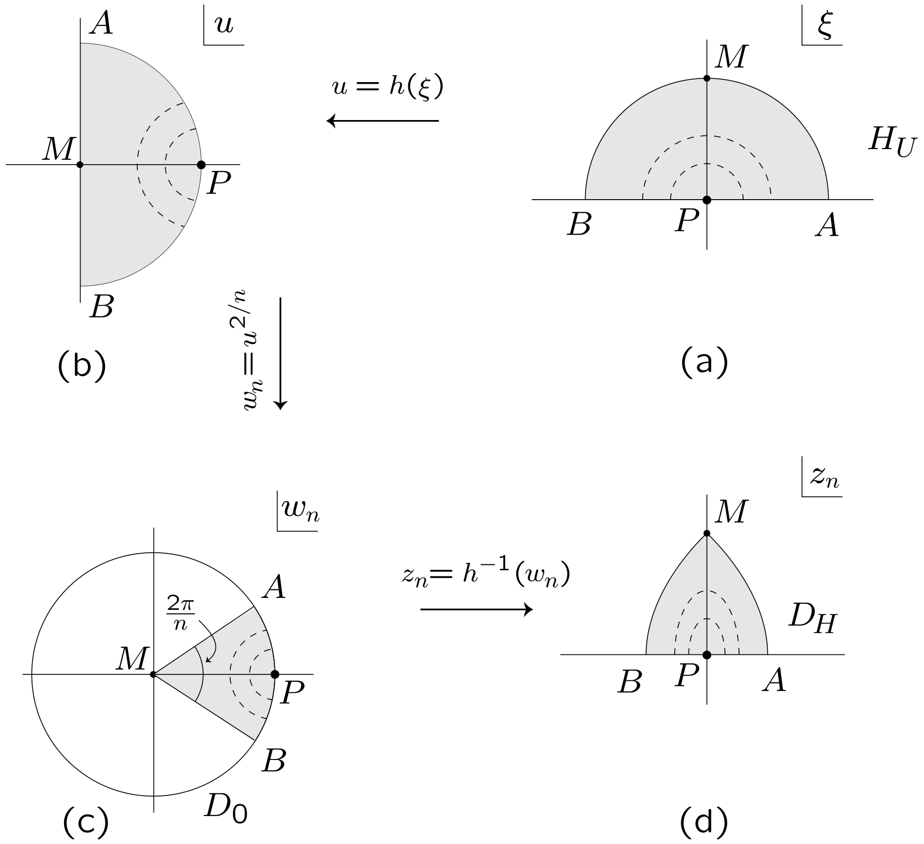

Figure 3: The definition of wedge states for arbitrary n. (a) The canonical half disk H_U. (b) The map of H_U into a vertical half-disk. (c) The inclusion of the vertical half-disk of (b) into the disk D_0 with global coordinate w. (d) The map of D_0 into the upper half plane D_H.

angle at $w_n = 0$ equal to $2\pi/n$. Figure 3c shows the disk D_0 in the w_n-plane with the puncture and the local coordinate. For any fixed n we call the $\langle n|$ the resulting surface state. Thus we have

$$\langle n|\phi\rangle \equiv \langle \check{f}_n \circ \phi(0)\rangle_{D_0} \qquad \forall \phi\,. \tag{3.13}$$

The state obtained when $n = 1$ is the identity state (see figure 4a). For this state the local coordinate patch in the w_n plane covers the full unit disk D_0 with a cut on the negative real axis. The left-half and the right-half of the string coincide along this cut. The state $n = 2$ is the vacuum state. In this case the image of H_U covers the right half of the full unit disk D_0 in the w_n plane. In the $n \to \infty$ limit, the image of H_U in the w_n coordinate is a 'thin sliver' of the disk D_0 (figure 4b). It was seen in [11] that the limit $n \to \infty$ of $\langle n|$ gives rise to a well-defined state. In the next subsection we shall give a detailed explanation for this result. This surface state $\langle \Xi|$, called the sliver, has the property that the left-half and the right-half of the string are as far as they can be on the unit disk.

Next we describe the state $|n\rangle$ using the coordinate z on the upper half plane (see figure 3*d*). We have

$$z_n = h^{-1}(w_n) = i\,\frac{1-w_n}{1+w_n} = \tan\Big(-\frac{i}{2}\ln w_n\Big)\,. \tag{3.14}$$

The composition of (3.12) and (3.14) gives [11]

$$z_n = \tan\Big(\frac{2}{n}\tan^{-1}(\xi)\Big) \equiv \widetilde{f}_n(\xi)\,, \tag{3.15}$$

and we have

$$\langle n|\phi\rangle = \langle \widetilde{f}_n \circ \phi(0)\rangle_{D_H}\,. \tag{3.16}$$

Finally we introduce the coordinate $\widehat{w}$ for the presentation of the surface state $|n\rangle$ using a disk $\widehat{D}$. Using eqs. (3.9) and (3.11) we see that

$$\widehat{w}_n = (w_n)^{n/2}\,. \tag{3.17}$$

This is simply the map inverse to that taking figure 3*b* to figure 3*c*, but extended for all of D_0. Under this map the unit disk D_0 in the w_n-coordinates is mapped to a cone in the $\widehat{w}_n$ coordinate, subtending an angle $n\pi$ at the origin $\widehat{w}_n = 0$. We shall denote this cone by $\widehat{D}_n$. Thus we have

$$\langle n|\phi\rangle = \langle h \circ \phi(0)\rangle_{\widehat{D}_n}\,. \tag{3.18}$$

In the $n \to \infty$ limit $\widehat{D}_n$ can be viewed as an infinite helix, as shown in figure 5.

3.3 Using SL(2, ℝ)-to resolve singular coordinates

All three descriptions of the sliver, using eqs. (3.13), (3.16) and (3.18), are apparently singular. (3.13) and (3.16) are singular since the corresponding maps $\check{f}_n(\xi)$ and $\widetilde{f}_n(\xi)$, defined in eqs. (3.11) and (3.15) are singular in the $n \to \infty$ limit. On the other hand, (3.18) is singular since the region $\widehat{D}_n$ becomes a cone with infinite excess angle at the origin in the $n \to \infty$ limit. We shall now show that these are only apparent singularities, and that for any Fock space state $|\phi\rangle$, $\langle\Xi|\phi\rangle$ is a perfectly finite number.

The main ingredient of the proof will be the SL(2, ℝ) invariance of the correlation functions in the upper half plane. Given any SL(2, ℝ) map $R(z)$, we have the relation:

$$\Big\langle \prod_i \mathcal{O}_i(x_i)\Big\rangle_{D_H} = \Big\langle \prod_i R\circ\mathcal{O}_i(x_i)\Big\rangle_{D_H}\,, \tag{3.19}$$

for any set of operators $\mathcal{O}_i$ and with D_H denoting the upper half plane. Thus we can rewrite eq. (3.16) as:

$$\langle n|\phi\rangle = \langle R_n \circ \widetilde{f}_n \circ \phi(0)\rangle_{D_H}\,, \tag{3.20}$$

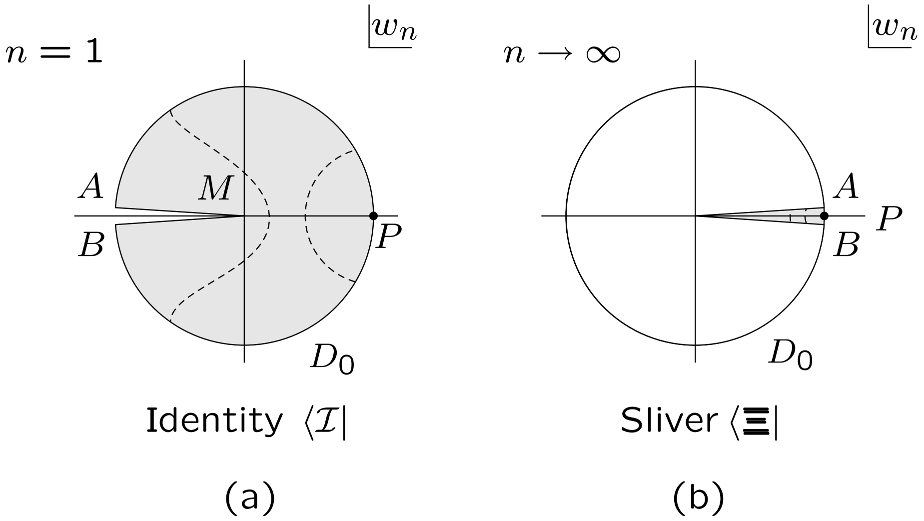

Figure 4: (a) The surface state corresponding to the identity string field $\langle \mathcal{I}|$. Here the image of H_U covers the full disk, except for a cut in the negative real axis. (b) The surfaces state corresponding to the sliver $\langle \Xi|$. Here the image of H_U covers an infinitesimally thin sliver around the positive real axis.

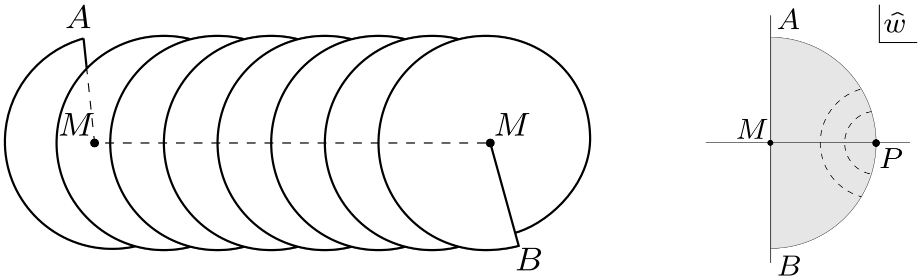

Figure 5: Using $\widehat{w}$ global coordinates the sliver appears as a cone with infinite excess angle– namely, an infinite helix. The segments AM and BM represent the left-half and the right-half of the string. The local coordinate patch, represented by the shaded half disk shown to the right, must be glued in to form the complete surface.

for any set of $\mathrm{SL}(2,\mathbb{R})$ maps R_n. Thus if we can find a sequence of maps R_n such that $R_n \circ \widetilde{f}_n$ approaches a finite limit f as $n \to \infty$ with $f(\xi)$ non-singular at the origin, then we can define the sliver $\langle \Xi|$ through the relations:

$$\langle \Xi|\phi\rangle = \langle f \circ \phi(0)\rangle_{D_H} . \tag{3.21}$$

In this case we choose:

$$R_n(z) = \frac{n}{2} z \,, \tag{3.22}$$

so that

$$\begin{aligned} f(\xi) &= \lim_{n\to\infty} R_n \circ \widetilde{f}_n(\xi) \\ &= \lim_{n\to\infty} \frac{n}{2} \tan\left(\frac{2}{n}\tan^{-1}(\xi)\right) = \tan^{-1}\xi \,. \end{aligned} \tag{3.23}$$

Since this map is non-singular at $\xi = 0$, this provides a finite answer for $\langle \Xi | \phi \rangle$ for any Fock space state $|\phi\rangle$, thereby providing a non-singular description of the sliver. We shall denote by

$$z' = f(\xi)\,, \tag{3.24}$$

the new coordinate on D_H. This picture of the sliver state is shown in figure 6a,b. The local coordinate patch has become the full strip bounded by the lines $\mathrm{Re}(z') = \pm\pi/4$. These two semiinfinite lines are the left- and right- halves of the string. They meet at the midpoint M which has been sent to $z' = i\infty$. This is actually the only reminder that the sliver state is a limit of a sequence of fully regular states. While the coordinate at the puncture can be taken to be regular at this limit, and thus the state $\langle \Xi |$ is well-defined, in this limit the map of H_U into the disk fails to be regular at one point. The string midpoint has been sent to the boundary of the disk. While this does not affect any computation involving Fock space states, this fact is significant in that it shows that *there is no fully regular geometric presentation of the sliver*. This might cause some worry since, as mentioned before, for gluing purposes the image of the arc $\{|\xi| = 1,\ 0 < \arg(\xi) < \pi\}$ should be inside the disk. We shall see, however, that gluing operations involving the sliver can be made well-defined despite this singularity.

This description of the sliver can be obtained by considering another sequence of maps. For this we introduce a new coordinate system

$$\widehat{z}_n = \frac{1}{2i} \ln \widehat{w}_n\,. \tag{3.25}$$

The cone $\widehat{D}_n$ in the $\widehat{w}_n$ coordinate system maps to a semiinfinite cylinder $\widehat{C}_n$ in the $\widehat{z}_n$ coordinate system with $\widehat{z}_n$ spanning the range:

$$-\frac{\pi}{4} \le \mathrm{Re}(\widehat{z}_n) \le \left(\frac{n}{2} - \frac{1}{4}\right)\pi\,, \qquad \mathrm{Im}(\widehat{z}_n) \ge 0\,, \qquad \widehat{z}_n \simeq \widehat{z}_n + n\,\frac{\pi}{2}\,. \tag{3.26}$$

The local coordinate patch is the region:

$$-\frac{\pi}{4} \le \mathrm{Re}(\widehat{z}_n) \le \frac{\pi}{4}\,, \qquad \mathrm{Im}(\widehat{z}_n) \ge 0\,. \tag{3.27}$$

This has been shown in figure 7. The relationship between $\widehat{z}_n$ and the local coordinate ξ follows from eqs. (3.9) and (3.25):

$$\widehat{z}_n = \tan^{-1}\xi \equiv f(\xi)\,. \tag{3.28}$$

Thus we have

$$\langle n | \phi \rangle = \langle f \circ \phi(0) \rangle_{\widehat{C}_n} \quad \forall |\phi\rangle \in \mathcal{H}\,. \tag{3.29}$$

Note now that using the periodicity along the $\mathrm{Re}(\widehat{z}_n)$ direction we could take the range of $\mathrm{Re}(\widehat{z}_n)$ to be $-n\pi/4 \le \mathrm{Re}(\widehat{z}_n) \le n\pi/4$. In this case as $n \to \infty$, $\widehat{C}_n$ approaches the full UHP and the coordinate $\widehat{z}_n$ approaches the coordinate z' introduced earlier.

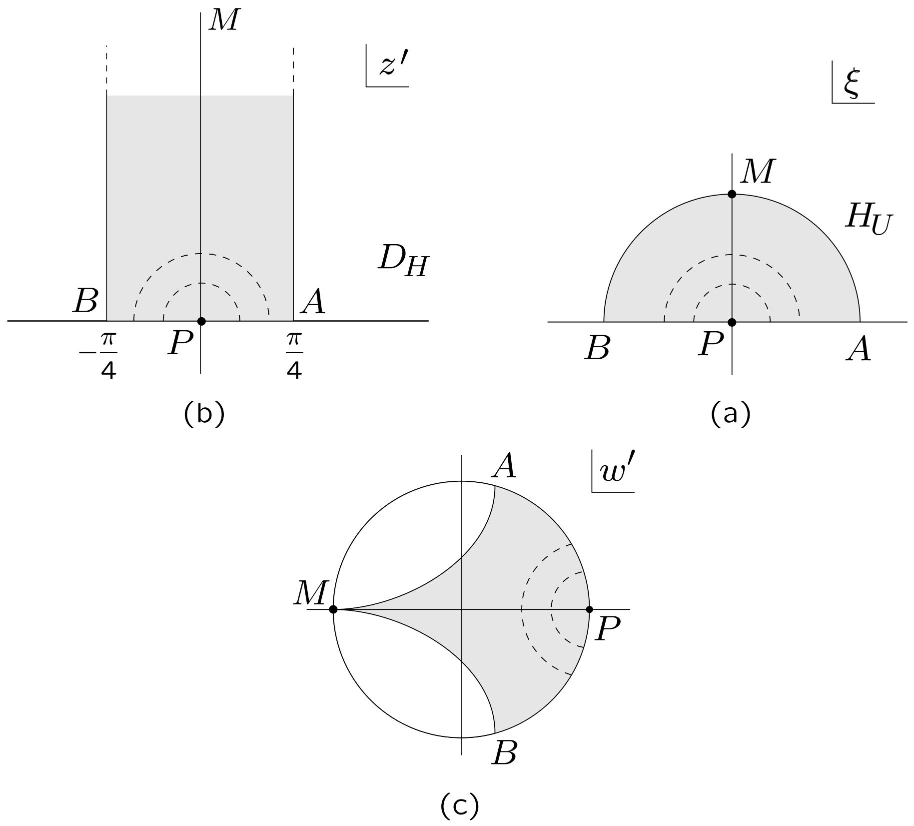

Figure 6: The regular presentations of the sliver surface state with non-singular coordinates at the puncture. In (b) the disk D is the full upper half plane, and the image of the local coordinate disk H_U is the vertical strip whose boundaries are the left-half and right-half of the string. The string midpoint is at infinity. (c) Mapping D_H back to a unit disk with $w' = h(z')$ we find a regular presentation on the disk.

Having obtained a regular description of the sliver we can map it back to the unit disk by writing $w' = h(z')$, with h defined in (3.6). This gives us figure 6c; the regular presentation in the disk. The only remnant of the singular origin is the fact that the open string midpoint reaches the boundary of the disk and the open string develops a cusp at the midpoint. As a result of this, at least intuitively, the surface representing the sliver appears to be cut into two disjoint pieces by the local coordinate patch. One could therefore expect the wavefunctional of the sliver to factorize into two wavefunctionals, one corresponding to the data on the left-half of the string, and the other to the data on the right-half of the string. It was seen in refs. [31, 49] that this intuitive expectation is indeed realized.

We would like to emphasize that the use of $\mathrm{SL}(2,\mathbb{R})$ invariance to resolve singular coordinates is most effective for once punctured disks. For the case of three punctured disks, the three (distinguishable) punctures can always be fixed at three points and there are no $\mathrm{SL}(2,\mathbb{R})$ invariances left. If the local coordinate at one puncture is singular, nothing can be done about it. This is not the case for once punctured

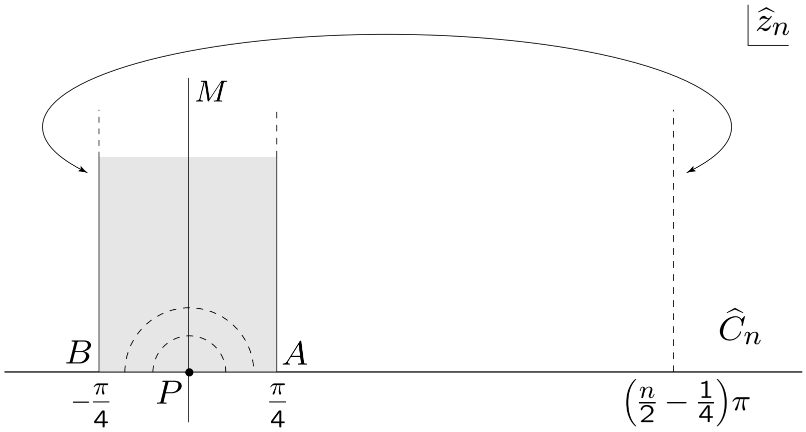

Figure 7: The finite n approximation to the sliver presented in the $\widehat{z}_n$ coordinate.

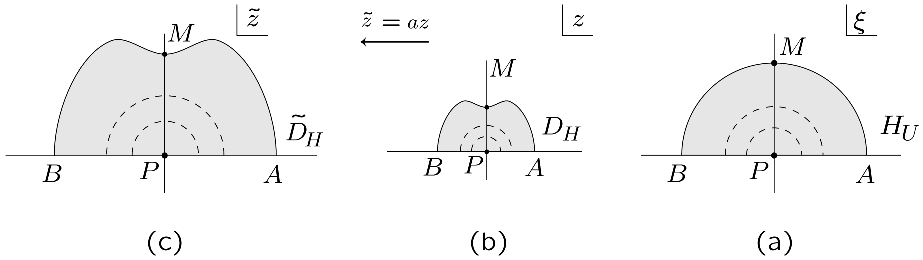

Figure 8: The power of conformal isometries in resolving singular coordinates on once-punctured disks is illustrated by showing two versions of a disk (D_H and $\tilde{D}_H$) that, while looking different, represent the same Riemann surface. Using an $a \to \infty$ limit one could resolve a vanishingly small image of H_U in D_H.

disks, because after fixing the position of the puncture there are powerful conformal isometries left over.[5] For example, after fixing the puncture at $z = 0$, the maps $\tilde{z} = az/(cz+d)$ do change the looks of the local coordinate as shown in figure 8. Even though the local coordinate patch in the z or $\tilde{z}$ upper half planes look different, and even though the functions $\tilde{z} = \tilde{g}(\xi)$ and $z = g(\xi)$ are different, as Riemann surfaces with local coordinates they are indeed *identical*. For the case of two punctured disks, with punctures fixed at $z = 0$ and $z = 1$, for example, the maps $\tilde{z} = az/(z + (a-1))$ preserve the punctures. Near $z = 0$ the map looks like $\tilde{z} = az/(a-1)$ and near $z = 1$ it looks like $\tilde{z} = 1 + (z-1)(a-1)/a$. Since the scaling factors are inverses of each other, if only one coordinate is singular it cannot be resolved. Only particular cases when the coordinates are singular in a related way could be resolved.

3.4 Star multiplication of surface states

The star multiplication of two surface states is easiest to describe by representing them in the $\widehat{w}$ coordinate system. Let us assume that we have two such surface states,

[5]This of course does not mean that any singularity of the local coordinate patch can be removed by such transformations, but there is a reasonable degree of freedom.

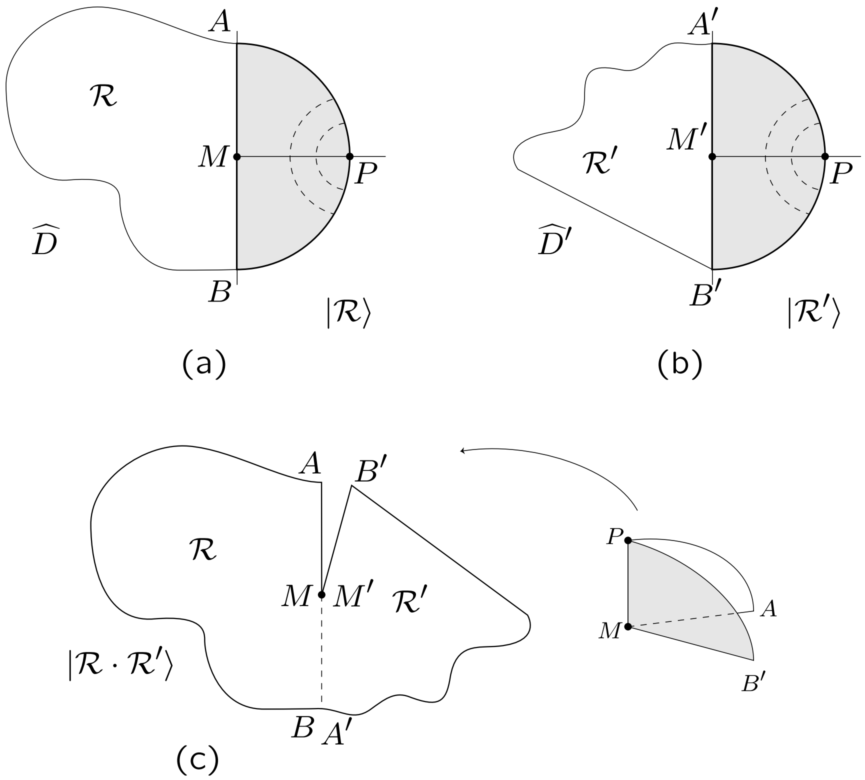

Figure 9: Star multiplication of surface states is simple if the states are presented with $\widehat{w}$ coordinates, in which case the local coordinate appears as a (shaded) half-disk in the disk $\widehat{D}$. Here are shown two surface states; $|\mathcal{R}\rangle$ in (a), and $|\mathcal{R}'\rangle$ in (b). The star product is shown in (c) and is obtained gluing the regions $\mathcal{R}$ and $\mathcal{R}'$ as indicated, and attaching the half disk (shown to the right) representing the local coordinate and the puncture.

and associated with them are the regions $\widehat{D}$ and $\widehat{D}'$, which describe the images of the disk in the corresponding $\widehat{w}$ coordinate system. Each of these disks contain as a subspace the region $h(H_U)$, — the local coordinate patch. We shall denote by $\mathcal{R}$ and $\mathcal{R}'$ the complement of $h(H_U)$ in $\widehat{D}$ and $\widehat{D}'$ respectively. By an abuse of notation we shall denote the surface states associated with the disks $\widehat{D}$ and $\widehat{D}'$ by $|\mathcal{R}\rangle$ and $|\mathcal{R}'\rangle$ respectively. This is represented diagrammatically in figure 9, where in parts (a) and (b) we show two surface states $|\mathcal{R}\rangle$ and $|\mathcal{R}'\rangle$ built in this way. The regions $h(H_U)$ have been shown shaded in this figure. In fact, once we have made it clear that the local coordinate is presented in the specific fashion chosen here, we could simply represent pictorially the surfaces as only the regions $\mathcal{R}$, namely the image of the full disk minus the local coordinate patch.[6]

If the surface states are presented this way, the star multiplication $|\mathcal{R}\rangle * |\mathcal{R}'\rangle$ is readily performed. One removes the local coordinate patches from the disks $\widehat{D}$

[6]Leaving off the local coordinate, of course, carries a small risk of confusion, especially because the local coordinate patch could be presented in other canonical ways, for example, as semi-infinite strips.

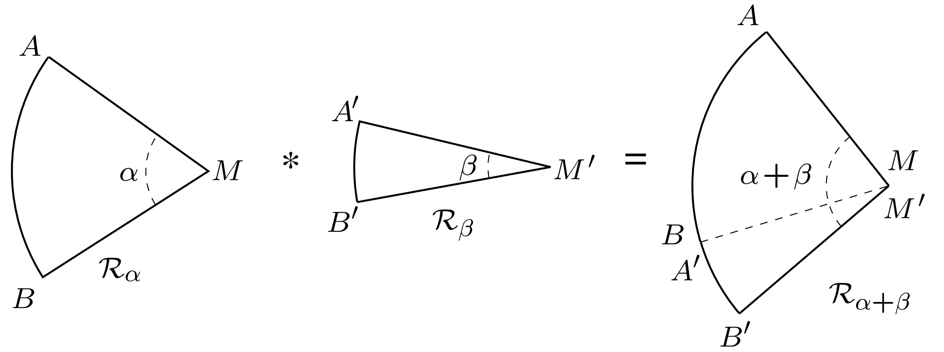

Figure 10: The star multiplication of a sector state with angle α to a sector state with angle β gives a sector state with angle $\alpha + \beta$. Sector states are just another presentation of wedge states.

and $\widehat{D}'$, and glues the right half (as viewed from the region of $\widehat{D}$ *outside* the local coordinate patch) MB of the $\mathcal{R}$ open string to the left half $A'M'$ of the $\mathcal{R}'$ open string. The result is the composite region $\mathcal{R} \cdot \mathcal{R}'$ shown in the figure. This region represents the star product of the states, namely

$$|\mathcal{R}\rangle * |\mathcal{R}'\rangle = |\mathcal{R} \cdot \mathcal{R}'\rangle \,. \tag{3.30}$$

In this region, the string is AMB', with AM the left half, and MB' the right half. The local coordinate patch shown to the right, must be glued in to produce the full picture of the surface state.

We now discuss the multiplication of wedge states $|n\rangle$. As we have already discussed, in the $\widehat{w}$ coordinate system relevant for the state $\langle n|$, the disk becomes a cone subtending an angle πn at the origin. If we remove the local coordinate patch the left over region becomes a sector of angle $\pi(n-1)$. If we denote by $|\mathcal{R}_\alpha\rangle$ a *sector state* arising from a sector of angle α, we have the identification of sector states with wedge states:

$$|n\rangle = |\mathcal{R}_{\pi(n-1)}\rangle \,. \tag{3.31}$$

Moreover, it is clear from our discussion above that star multiplication of sector states simply gives a sector whose angle is the sum of those in the product:

$$|\mathcal{R}_\alpha\rangle * |\mathcal{R}_\beta\rangle = |\mathcal{R}_{\alpha+\beta}\rangle \,, \tag{3.32}$$

This is illustrated in figure 10. Indeed the multiplication rule (3.32) is consistent with $|n\rangle * |m\rangle = |n + m - 1\rangle$, which is the familiar multiplication rule for wedge states.[7] The sliver arises from $n \to \infty$ and therefore corresponds to a sector state

[7]We hope the reader will not be confused by the dual use of the term surface state. In the *wedge states*, the focus is on the local coordinate which looks like a wedge inside a unit disk, in the *sector states*, the focus is on the complement of the local coordinates in the disk when the local coordinate patch is presented as a half-disk. This complement looks like a sector.

with infinite angle! This state was shown schematically in Figure 5. Indeed, it should be noted that among sector states only two can multiply to themselves, according to (3.32). One is the zero-angle sector state $|\mathcal{R}_0\rangle$ which is the identity $|\mathcal{I}\rangle$ (or the wedge state $|n = 1\rangle$). The other is the sliver, which can be viewed as the zero angle wedge state, or as the infinite angle sector state. Indeed, from (3.32) we have $|\mathcal{R}_\infty\rangle * |\mathcal{R}_\infty\rangle = |\mathcal{R}_\infty\rangle$.

For later use it will be useful to work out the precise relationship between the different coordinate systems appearing in the description of the product state $|\mathcal{R}_{\alpha+\beta}\rangle$ and the states $|\mathcal{R}_\alpha\rangle$ and $|\mathcal{R}_\beta\rangle$. Again this is simple in the $\widehat{w}$ coordinate system. For this let us take

$$\alpha = \pi(m-1)\,, \qquad \beta = \pi(n-1)\,, \qquad \alpha+\beta = \pi(m+n-2)\,. \tag{3.33}$$

If we denote by $\widehat{w}_m$, $\widehat{w}_n$ and $\widehat{w}_{m+n-1}$ the $\widehat{w}$ coordinates associated with the sector states $|\mathcal{R}_\alpha\rangle$, $|\mathcal{R}_\beta\rangle$ and $|\mathcal{R}_{\alpha+\beta}\rangle$ respectively, we have

$$\widehat{w}_{m+n-1} = \begin{cases} \widehat{w}_m & \text{in } \mathcal{R}_\alpha \\ e^{i\alpha}\widehat{w}_n & \text{in } \mathcal{R}_\beta . \end{cases} \tag{3.34}$$

In the $\widehat{z}_n$ coordinate system introduced in eq. (3.25) the gluing relations (3.34) take a very simple form:

$$\widehat{z}_{m+n-1} = \begin{cases} \widehat{z}_m & \text{for } \dfrac{\pi}{4} \leq \mathrm{Re}(\widehat{z}_m) \leq \left(\dfrac{m}{2} - \dfrac{1}{4}\right)\pi, \\[2ex] \widehat{z}_n + \dfrac{1}{2}(m-1)\pi & \text{for } \dfrac{\pi}{4} \leq \mathrm{Re}(\widehat{z}_n) \leq \left(\dfrac{n}{2} - \dfrac{1}{4}\right)\pi. \end{cases} \tag{3.35}$$

4. General D-brane solutions

In this section we shall give deformations of the sliver to generate new solutions of the equations of motion representing D-branes described by general BCFT's. The general idea is simple. We denote by BCFT_0 the *reference* BCFT in whose Hilbert space the string field takes value. The sliver of BCFT_0, whose matter part represents the D-brane solution associated with BCFT_0, is the surface state described in the previous section, with the specific boundary condition corresponding to BCFT_0 on the boundary of the surface. To get a solution representing the single D-brane of some other boundary conformal field theory BCFT' we must represent the sliver of BCFT' on the state space of BCFT_0, as we now explain. The construction assumes that BCFT_0 and BCFT' have the same bulk conformal field theory, but of course, differ in their boundary interactions.

Usually the sliver Ξ' of BCFT' will be described in the same manner as discussed in the last section, with all the correlation functions now being calculated in BCFT'. This, however, would express Ξ' as a state in the Hilbert space $\mathcal{H}'$ of BCFT', since in eq. (3.21), for example, ϕ will now represent a vertex operator of BCFT$'$. In order to express Ξ' in the Hilbert space of BCFT$_0$, we adopt the following procedure. As discussed in section 3.2, at an intuitive level, the wave-functional for $\langle\Xi'|$ is a functional of $\varphi_0(\sigma)$ with $\sigma = \frac{1}{i}\ln\xi$ labeling the coordinate along the string, obtained by functional integration over the two dimensional fields φ on the full disk (UHP in this case) minus the local coordinate patch, with boundary condition $\varphi = \varphi_0(\sigma)$ on the boundaries $\mathrm{Re}(z') = \pm\pi/4$, and the boundary condition appropriate to BCFT$'$ on the rest of the boundary of this region (z' real, $|\mathrm{Re}(z')| \geq \pi/4$). On the other hand given a state $|\phi\rangle$ in the Hilbert space $\mathcal{H}$ of BCFT$_0$, we represent the wave-functional of $|\phi\rangle$ as a functional of $\varphi_0(\sigma)$, obtained by performing path integral over φ in the inside of the local coordinate patch *with boundary condition appropriate for BCFT$_0$* on the real axis, the vertex operator ϕ of BCFT$_0$ inserted at the origin in the local coordinate system ξ, and the boundary condition $\varphi = \varphi_0(\sigma)$ on the semi-circle. Finally in order to calculate $\langle\Xi'|\phi\rangle$ we take the product of the wave-functional of $\langle\Xi'|$ and the wave-functional of $|\phi\rangle$ and integrate over $\varphi_0(\sigma)$. The result will be a functional integral over the fields φ on the full UHP, with boundary condition appropriate to BCFT$'$ in the range $|\mathrm{Re}(z')| \geq \pi/4$, boundary condition corresponding to BCFT$_0$ for $|\mathrm{Re}(z')| \leq \pi/4$ and the vertex operator ϕ inserted at the origin *in the local coordinate system*. This can be expressed as

$$\langle\Xi'|\phi\rangle = \langle f\circ\phi(0)\rangle'' \,, \tag{4.1}$$

where $\langle\ \rangle''$ denotes correlation function in a theory where we have BCFT$'$ boundary condition for $|\mathrm{Re}(z')| \geq \pi/4$ and BCFT$_0$ boundary condition for $|\mathrm{Re}(z')| \leq \pi/4$. ϕ is a vertex operator in BCFT$_0$ and $f(\xi) = \tan^{-1}\xi$ as usual.

In what follows, we shall show that after appropriate ultraviolet regularization, Ξ' defined this way squares to itself under $*$-multiplication, and also has the right tension for describing a D-brane associated to BCFT$'$. We will begin by considering this general case in the next subsection. Then we will discuss the situation where BCFT$'$ is replaced by a general two dimensional field theory obtained from BCFT$_0$ by some boundary perturbation.

4.1 Solution describing an arbitrary D-brane

We shall now describe the construction of the solution of the SFT equations of motion describing a D-brane corresponding to an arbitrary boundary conformal field theory BCFT$'$ with the same bulk CFT as BCFT$_0$. We start with the definition of Ξ' given in eq. (4.1). The effect of the change of the boundary condition beyond $|x'| \geq \pi/4$ can be represented by inserting boundary condition changing vertex operator $\sigma^\pm$ (discussed e.g. in [52, 53]) at $x' = \pm\pi/4$. In other words we can express (4.1) as

$$\langle\Xi'|\phi\rangle = \left\langle f\circ\phi(0)\,\sigma^+\left(\frac{\pi}{4}\right)\sigma^-\left(-\frac{\pi}{4}\right)\right\rangle . \tag{4.2}$$

If we denote by D and D$'$ the D-branes associated with BCFT$_0$ and BCFT$'$ respectively, then σ^+ denotes the vertex operator for the ground state of an open string whose left end (viewed from inside the UHP) is on the D$'$-brane and right end is on the D-brane, whereas σ^- denotes the vertex operator for the ground state of open string whose left end is on the D-brane and right end is on the D$'$-brane. In anticipation of short-distance divergences, we shall actually put σ^- and σ^+ at $(-\pi/4-\epsilon)$ and $(\pi/4+\epsilon)$ respectively, where ϵ is a small positive number. We shall also use the description of the sliver as limit of finite n wedge states in the $\widehat{z}_n$ coordinate introduced in eq. (3.25). Thus we have

$$\langle \Xi' | \phi \rangle = \lim_{n\to\infty} \Big\langle f\circ\phi(0)\, \sigma^+\Big(\frac{\pi}{4}+\epsilon\Big)\sigma^-\Big(\frac{n}{2}\pi - \frac{\pi}{4} - \epsilon\Big)\Big\rangle_{\widehat{C}_n} . \tag{4.3}$$

We now calculate $\Xi' * \Xi'$. From the gluing relations (3.35) we get,

$$\begin{aligned}\langle \Xi' * \Xi' | \phi \rangle = \lim_{m,n\to\infty} \Big\langle & f\circ\phi(0)\, \sigma^+\Big(\frac{\pi}{4}+\epsilon\Big)\, \sigma^-\Big(\Big(\frac{m}{2}-\frac{1}{4}\Big)\pi - \epsilon\Big) \times \\ & \times \sigma^+\Big(\Big(\frac{m}{2}-\frac{1}{4}\Big)\pi + \epsilon\Big)\sigma^-\Big(\frac{1}{2}(m+n-1)\pi - \frac{\pi}{4} - \epsilon\Big)\Big\rangle_{\widehat{C}_{m+n-1}} .\end{aligned} \tag{4.4}$$

Thus the $\sigma^\pm$ at $\frac{1}{2}(m-\frac{1}{2})\pi \pm \epsilon$ come close as $\epsilon \to 0$ and give rise to a divergent factor $(2\epsilon)^{-2h}$ where h is the conformal weight of $\sigma^\pm$. Hence we have $\Xi' * \Xi' = (2\epsilon)^{-2h}\Xi'$. This requires us to redefine Ξ' by absorbing a factor of $(2\epsilon)^{2h}$, so that it squares to itself under $*$-multiplication

$$\Xi'_R \equiv (2\epsilon)^{2h}\; \Xi' \quad \to \quad \Xi'_R * \Xi'_R = \Xi'_R . \tag{4.5}$$

We note, however, that even for finite ϵ, the state Ξ'_R still squares to itself. Indeed, the product $\sigma^+(\frac{1}{2}(m-\frac{1}{2})\pi+\epsilon)\sigma^-(\frac{1}{2}(m-\frac{1}{2})\pi-\epsilon)$ can be expanded in an operator product expansion, and since these operators are moved to ∞ in the $m, n \to \infty$ limit, only the identity operator in this operator product expansion would contribute. The coefficient of the identity operator is given by $(2\epsilon)^{-2h}$ even for finite ϵ.

Since BCFT$_0$ and BCFT$'$ differ only in the matter sector, it is clear that $|\Xi'_R\rangle$ has the factorized form

$$|\Xi'_R\rangle = |\Xi_g\rangle \otimes |\Xi'_{R,m}\rangle , \tag{4.6}$$

where Ξ_g is a universal ghost factor. Normalizing Ξ_g (which is independent of the choice of BCFT$'$) such that $\Xi_g *^g \Xi_g = \Xi_g$, we can ensure that

$$\Xi'_{R,m} *^m \Xi'_{R,m} = \Xi'_{R,m} . \tag{4.7}$$

Thus we can now construct a new D-brane solution by taking the product $|\Psi_g\rangle \otimes |\Xi'_{R,m}\rangle$, where $|\Psi_g\rangle$ is the same universal ghost state that appears in the construction of the D-brane solution corresponding to BCFT$_0$.

We shall now calculate the tension associated with this new D-brane solution. For this we need to compute $\langle \Xi'_{R,m} | \Xi'_{R,m} \rangle_m$, where the subscript m denotes matter. We have,

$$\langle \Xi'_{R,m} | \Xi'_{R,m} \rangle_m = (2\epsilon)^{4h} \, \langle \Xi'_m | \Xi'_m \rangle_m \, . \tag{4.8}$$

Calculation of $\langle \Xi'_m | \Xi'_m \rangle_m$ is again simple in the $\widehat{z}_n$ coordinate system. We first compute the $*$-product of the two states, and then in the final glued surface with coordinate $\widehat{z}_{m+n-1}$ we remove the local coordinate patch $-\pi/4 \le \mathrm{Re}(\widehat{z}_{m+n-1}) \le \pi/4$ and identify the lines $\mathrm{Re}(\widehat{z}_{m+n-1}) = \pm\pi/4$. This produces the semi-infinite cylinder $\widetilde{C}_{m+n-2}$ defined by $\frac{\pi}{4} \le \mathrm{Re}(z_{m+n-2}) \le \frac{\pi}{4} + (m+n-2)\frac{\pi}{2}$ and $\mathrm{Im}(z_{m+n-2}) \ge 0$. We therefore find

$$\begin{aligned}\langle \Xi'_m | \Xi'_m \rangle_m = \lim_{m,n\to\infty} \Big\langle \sigma^+\Big(\frac{\pi}{4}+\epsilon\Big) \, \sigma^-\Big(\Big(\frac{m}{2}-\frac{1}{4}\Big)\pi - \epsilon\Big) \times & \qquad (4.9)\\ \times \, \sigma^+\Big(\Big(\frac{m}{2}-\frac{1}{4}\Big)\pi + \epsilon\Big) \sigma^-\Big(\frac{1}{2}(m+n-1)\pi - \frac{\pi}{4} - \epsilon\Big) \Big\rangle_{\widetilde{C}_{m+n-2}} , &\end{aligned}$$

where the correlation function is now being computed in the matter BCFT. We get a factor of $(2\epsilon)^{-2h}$ coming from $\sigma^\pm$ inserted at $(\frac{m}{2}-\frac{1}{4})\pi \pm \epsilon$ as before, but there is another factor of $(2\epsilon)^{-2h}$ coming from the other $\sigma^\pm$ insertions that happen at points separated by a (minimal) distance 2ϵ on the cylinder $\widetilde{C}_{m+n-2}$. These exactly cancel the explicit factor of $(2\epsilon)^{4h}$ in (4.8). Since from the definition of $\sigma^\pm$ it is clear that in the $\epsilon \to 0$ limit we have BCFT$'$ boundary condition on the full real z_{m+n-2} axis, we find that $\langle \Xi'_{R,m} | \Xi'_{R,m} \rangle_m$ is given by the partition function of the deformed boundary CFT on the $\widetilde{C}_{m+n-2}$ cylinder

$$\langle \Xi'_{R,m} | \Xi'_{R,m} \rangle_m = Z_{\widetilde{C}_{m+n-2}}(\mathrm{BCFT}') \sim Z_{D_0}(\mathrm{BCFT}') \, , \tag{4.10}$$

where in the last step we relate this partition function to the one on the standard unit disk. This is possible because of conformal invariance. Any constant multiplicative factor that might appear due to conformal anomaly depends only on the bulk central charge and is independent of the choice of BCFT$'$. This can at most give rise to a universal multiplicative factor. Since the partition function of BCFT$'$ on the unit disk is proportional to the tension of the corresponding D-brane [34]–[38], we see that the tension $\langle \Xi'_{R,m} | \Xi'_{R,m} \rangle_m$ computed from vacuum string field theory agrees with the known tension of the BCFT$'$ D-brane, up to an overall constant factor independent of BCFT$'$.

Arguments similar to the one given for $\Xi' * \Xi'$ show that the result (4.10) holds even when ϵ is finite. In this case we have two pairs of $\sigma^\pm$ on the boundary, with the first pair being infinite distance away from the second pair. Thus we can expand each pair using operator product expansion and only the identity operator contributes, giving us back the partition function of BCFT$'$ on the disk. From this we see that we have a one parameter family of solutions, labeled by ϵ, describing the same D-brane.

We expect these solutions to be related by gauge transformations, since $\partial_\epsilon \Xi'_{R,m}$ has finite norm (as can be easily verified) and hence is pure gauge according to the arguments to be given in section 5.

4.2 Multiple D-branes and coincident D-branes

We first consider the construction of a configuration containing various D-branes associated to different BCFT$'$s. To this end, we note that the star product $\Xi' * \Xi$ of the BCFT$'$ solution and the BCFT$_0$ solution vanishes. Indeed, using the same methods as in the previous subsection, the computation of $\Xi' * \Xi$ leads to the cylinder $\widehat{C}_{m+n-1}$ with a σ^+ insertion at $\pi/4+\epsilon$ and a σ^- insertion at $(\frac{m}{2}-\frac{1}{4})-\epsilon$. In the $m, n \to \infty$ limit, σ^- moves off to infinity and as a result the correlation function vanishes since σ^- has dimension larger than zero *as long as BCFT$_0$ and BCFT$'$ are different.* Similar arguments show that $\Xi * \Xi'$ and $\langle \Xi_m | \Xi'_m \rangle_m$ also vanish. Thus the matter part of $\Xi + \Xi'_R$ is a new solution describing the superposition of the D-branes corresponding to BCFT$_0$ and BCFT$'$. Since no special assumptions were made about BCFT$_0$ nor BCFT$'$, it follows that $\Xi' * \Xi'' = \Xi'' * \Xi' = 0$ and $\langle \Xi'_m | \Xi''_m \rangle_m = 0$ for any two different BCFT$'$ and BCFT$''$, and hence we can superpose any number of slivers to form a solution. This in particular also includes theories which differ from each other by a small marginal deformation. Special cases of this phenomenon, in the case of D-branes in flat space-time, have been discussed in ref. [49].

This procedure, however, is not suitable for superposing D-branes associated with the same BCFT, i.e. for parallel coincident D-branes. For example, if we take BCFT$'$ to differ from BCFT$_0$ by an exactly marginal deformation with deformation parameter λ, then in the $\lambda \to 0$ limit the operators $\sigma^\pm$ both approach the identity operator (having vanishing conformal weight), and although the argument for the vanishing of $\Xi * \Xi'$ holds for any non-zero λ, it breaks down at $\lambda = 0$.

In order to construct a superposition of identical D-branes, one can proceed in a different way. First consider getting coincident BCFT$_0$ branes. To this end we introduce a modified BCFT$_0$ sliver

$$\langle \Xi_\chi | \phi \rangle = \lim_{n\to\infty} \Big\langle f \circ \phi(0)\, \chi^+\Big(\frac{\pi}{4}+\epsilon\Big) \chi^-\Big(\frac{n}{2}\pi - \frac{\pi}{4} - \epsilon\Big)\Big\rangle_{\widehat{C}_n} . \tag{4.11}$$

Here $\chi^\pm$ are a conjugate pair[8] of operators of BCFT$_0$, having a common dimension h greater than zero, and representing some excited states of the open string with *both ends having BCFT$_0$ boundary condition.* Thus, throughout the real line we have BCFT$_0$ boundary conditions. We require that the coefficient of the identity in the OPE $\chi^-(x)\chi^+(y)$ is given by $|x-y|^{-2h_i}$, and that this OPE does not contain any other operator of dimension ≤ 0.

[8]We need to choose $\chi^\pm$ to be conjugates of each other so that the string field is hermitian.

The clear parallel between eq. (4.11) and eq. (4.3), describing the BCFT$'$ D-brane, implies that an analysis identical to the one carried out in the previous section will show that:

1. This new state Ξ_χ (after suitable renormalization as in eq. (4.5)) squares to itself under $*$-multiplication.

2. The BPZ norm of the matter part of Ξ_χ is proportional to the partition function of BCFT$_0$ on the unit disk.

3. Ξ_χ has vanishing $*$-product with Ξ.

Thus the matter part of this state gives another representation of the D-brane associated with BCFT$_0$, and we can construct a pair of D-branes associated with BCFT$_0$ by superposing the matter parts of Ξ and Ξ_χ.

This construction can be easily generalized to describe multiple BCFT$_0$ D-branes. We construct different representations of the same D-brane by using different vertex operators $\chi^{(i)\pm}$ in BCFT$_0$ satisfying the 'orthonormality condition' that the coefficient of the identity operator in the OPE of $\chi^{(i)-}(x)\chi^{(j)+}(y)$ is given by $\delta_{ij}|x-y|^{-2h_i}$, and that this OPE does not contain any other operator of dimension ≤ 0. The correponding solutions $\Xi_{\chi^{(i)}}$ all have vanishing $*$-product with each other, and hence can be superposed to represent multiple D-branes associated with BCFT$_0$.

If instead we want to construct a superposition of identical BCFT$'$ D-branes, we need to replace $\sigma^\pm$ in eq. (4.3) by another pair of vertex operators $\tau^\pm$ which represent some excited states of open strings with one end satisfying BCFT$_0$ boundary condition and the other end satisfying BCFT$'$ boundary condition. The corresponding state Ξ'_τ will give another representation of the D-brane associated with BCFT$'$, having vanishing $*$-product with Ξ' as long as the operator product of σ^- with τ^+ does not contain the identity operator or any other operator of dimension ≤ 0. Hence we can superpose these solutions to construct new solutions.

This procedure of adding vertex operators near $\pm\pi/4$ to create new solutions representing the same D-brane is the BCFT version of the use of excited states of half-strings [31, 49] for the same purpose.

4.3 Finite deformations of the sliver

We shall now consider a class of solutions associated with the sliver for boundary field theories which are not necessarily conformal. Let us begin with the description of the sliver of BCFT$_0$ given in the $\widehat{z}_n$ coordinate system defined in eq. (3.25). We have from eq. (3.29):

$$\langle\Xi|\phi\rangle = \lim_{n\to\infty}\langle f\circ\phi(0)\rangle_{\widehat{C}_n} \qquad \forall\,|\phi\rangle\in\mathcal{H}\,, \tag{4.12}$$

with $f(\xi) = \tan^{-1}\xi$, and $\langle\cdot\rangle$ denotes correlation function calculated in BCFT_0. Now suppose V is a local vertex operator in the matter sector of BCFT_0. We define a new state $\langle\Xi^{V,\lambda}|$ through the relation:[9]

$$\langle\Xi^{V,\lambda}|\phi\rangle = \lim_{n\to\infty}\Big\langle \exp\Big(-\lambda\int_{\frac{\pi}{4}}^{(\frac{n}{2}-\frac{1}{4})\pi} V(x_n)dx_n\Big) f\circ\phi(0)\Big\rangle_{\widehat{C}_n} \qquad \forall\,|\phi\rangle\in\mathcal{H}\,, \quad (4.13)$$

where $x_n = \mathrm{Re}(\widehat{z}_n)$, λ is a constant, and the integration is done over the real $\widehat{z}_n$ axis *excluding the part that is inside the local coordinate patch.* The description of this state can be made more transparent by taking the full range of $\mathrm{Re}(\widehat{z}_n)$ to be $[-\frac{n}{4}\pi, \frac{n}{4}\pi]$ with the local coordinate patch in the region $-\pi/4 \le \mathrm{Re}(\widehat{z}_n) \le \pi/4$, and then identifying $\widehat{z}_n$ as the coordinate z' on the upper half plane in the $n\to\infty$ limit. This gives

$$\langle\Xi^{V,\lambda}|\phi\rangle = \left\langle \exp\left(-\lambda\int_{|x'|>\frac{\pi}{4}} V(x')\,dx'\right) f\circ\phi(0)\right\rangle_{D_H}\,, \quad (4.14)$$

where $x' = \mathrm{Re}(z')$. This expression should be treated as a correlation function in a theory where on part of the boundary we have the usual boundary action corresponding to BCFT_0, and on part of the boundary we have a modified boundary action obtained by adding the integral of V to the original action. In defining this we need to use suitable operator ordering, and regularization and renormalization prescriptions to remove the short distance singularities (see e.g. ref. [54] for a discussion of this). Typically this requires V to be of dimension ≤ 1. In special cases, as discussed in [41], we may also be able to include operators of higher dimensions, but as we shall see later these do not generate new solutions. We shall not explicitly take into account the effects of ultra-violet divergences in our analysis, and hence the results of this and the next subsection will be somewhat formal. The analysis, however, can be made concrete in specific examples, e.g. of the kind discussed in refs. [41]. If we regard the effect of λ deformation to be a change in the boundary condition, then (4.14) represents a correlation function with BCFT_0 boundary condition in the range $-\pi/4 \le x \le \pi/4$ and the modified boundary condition outside this range.

We shall now show that $|\Xi^{V,\lambda}\rangle$ satisfies the relation:

$$\Xi^{V,\lambda} * \Xi^{V,\lambda} = \Xi^{V,\lambda}\,. \quad (4.15)$$

To compute the $*$-product in the left hand side of the above equation, we use the $\widehat{z}_n$ coordinate system, keeping n finite and taking the $n\to\infty$ limit at the end. The advantage of using the $\widehat{z}_n$ coordinate system is in the simplicity of the gluing relations (3.35); since the coordinate $\widehat{z}_{m+n-1}$ of the surface after gluing is related to the coordinates $\widehat{z}_m$ and $\widehat{z}_n$ of the original surfaces by simple shift, there is no

[9] A construction that is similar in spirit but uses a different geometry was suggested in ref. [48].

conformal transformation of the factors $\int V(x_n)dx_n$ except for the appropriate change of range. Thus the result of computing $\Xi^{V,\lambda} * \Xi^{V,\lambda}$ is:

$$\langle \Xi^{V,\lambda} * \Xi^{V,\lambda}|\phi\rangle = \lim_{m,n\to\infty} \Big\langle \exp\Big(-\lambda \int_{\frac{\pi}{4}}^{(\frac{m}{2}-\frac{1}{4})\pi} V(x_{m+n-1})dx_{m+n-1}\Big) \times \tag{4.16}$$
$$\times \exp\Big(-\lambda \int_{(\frac{m}{2}-\frac{1}{4})\pi}^{(\frac{n}{2}-\frac{1}{4})\pi+\frac{1}{2}(m-1)\pi} V(x_{m+n-1})dx_{m+n-1}\Big) f\circ\phi(0)\Big\rangle_{\widehat{C}_{m+n-1}} .$$

This can be rewritten as

$$\langle \Xi^{V,\lambda} * \Xi^{V,\lambda}|\phi\rangle = \lim_{m,n\to\infty} \Big\langle \exp\Big(-\lambda \int_{\frac{\pi}{4}}^{\frac{m+n-1}{2}\pi-\frac{\pi}{4}} V(x_{m+n-1})dx_{m+n-1}\Big) f\circ\phi(0)\Big\rangle_{\widehat{C}_{m+n-1}} . \tag{4.17}$$

This is precisely eq. (4.13) with n replaced by $m+n-1$ (which is taken to infinity). This establishes the required result (4.15).

In composing the integrals in passing to (4.17) the insertions of V do coincide at one point, namely at $x_{m+n-1} = (\frac{m}{2}-\frac{1}{4})\pi$. This will cause additional divergences even if the exponentials appearing in the definition of each $\Xi^{V,\lambda}$ are ultraviolet regularized. This divergence can be regulated as in subsection 4.1 by defining $\Xi^{V,\lambda}$ through eq. (4.13), with the x_n integral running from $\pi/4+\epsilon$ to $(\frac{n}{2}-\frac{1}{4})\pi-\epsilon$. Doing so, we miss in the exponent of eq. (4.17) an integral of λV over the range $((\frac{m}{2}-\frac{1}{4})\pi-\epsilon) \le x_{m+n-1} \le ((\frac{m}{2}-\frac{1}{4})\pi+\epsilon)$ compared to the expression for $\langle \Xi^{V,\lambda}|\phi\rangle$. In the $m,n\to\infty$ limit this region moves off to infinity, and as a result contributes an ϵ dependent multiplicative factor to the correlation function given by the expectation value of the missing operator in the deformed theory.[10] This is the analog of the factor of $(2\epsilon)^{-2h}$ which arose in the analysis of subsection 4.1, and can be absorbed in the definition of $\Xi^{V,\lambda}$ to ensure that it squares to itself under $*$-multiplication.

Since the operator V is in the matter sector, $|\Xi^{V,\lambda}\rangle$ has the factorized form

$$|\Xi^{V,\lambda}\rangle = |\Xi_g\rangle \otimes |\Xi_m^{V,\lambda}\rangle\,, \tag{4.18}$$

where Ξ_g is independent of V and λ. As before, with $\Xi_g *^g \Xi_g = \Xi_g$ we ensure that

$$\Xi_m^{V,\lambda} *^m \Xi_m^{V,\lambda} = \Xi_m^{V,\lambda}\,. \tag{4.19}$$

Thus we can now construct new D-brane solutions by taking the product $|\Psi_g\rangle\otimes|\Xi_m^{V,\lambda}\rangle$, where $|\Psi_g\rangle$ is the universal ghost state that appears in the D25-brane solution.

[10] We are assuming here that the correlation function of the missing operator in the deformed theory satisfies cluster property.

4.4 Computation of the tension

We would now like to compute the tension associated with this solution. This is proportional to $\langle \Xi_m^{V,\lambda} | \Xi_m^{V,\lambda} \rangle_m$.[11] Computation of this inner product is again simple in the $\widehat{z}_n$ coordinates and the relevant geometry was discussed above (4.9). We therefore find

$$\langle \Xi_m^{V,\lambda} | \Xi_m^{V,\lambda} \rangle_m = \lim_{m,n\to\infty} \left\langle \exp\left(-\lambda \int_{\frac{\pi}{4}}^{\frac{m+n-1}{2}\pi - \frac{\pi}{4}} V(x_{m+n-2}) dx_{m+n-2} \right) \right\rangle_{\widetilde{C}_{m+n-2}} , \tag{4.20}$$

where $x_{m+n-2} = \mathrm{Re}(z_{m+n-2})$. If we do a more careful analysis using the regularized form of $\Xi^{V,\lambda}$, there will be two missing regions, each of width 2ϵ, in the exponent of eq. (4.20). These regions are infinite distance apart in the $m, n \to \infty$ limit and each of these regions give rise to an extra multiplicative factor, equal to the one that appeared in the computation of $\Xi^{V,\lambda} * \Xi^{V,\lambda}$, in eq. (4.20). These exactly cancel the ϵ-dependent normalization of $\Xi^{V,\lambda}$ required to ensure that the regularized $\Xi^{V,\lambda}$ squares to itself under $*$-multiplication. This is analogous to the corresponding phenomenon in section 4.1.

We now define a rescaled coordinate u as $u = 4(z_{m+n-2} - \frac{\pi}{4})/(m+n-2)$ so that $\mathrm{Re}(u)$ ranges from 0 to 2π. Thus in the u coordinate we have a semiinfinite cylinder C of circumference 2π. Writing $u = i\rho + \theta$, and taking into account the conformal transformation of the vertex operator V under this scale transformation, we get:

$$\langle \Xi_m^{V,\lambda} | \Xi_m^{V,\lambda} \rangle = \lim_{m,n\to\infty} \left\langle \exp\left(-\lambda_R \int_{\theta=0}^{2\pi} d\theta V_R(\theta) \right) \right\rangle_C , \tag{4.21}$$

where $\lambda_R \int V_R$ denotes the operator to which the perturbation $\lambda \int V$ flows under the rescaling by $(m+n-2)/4$. In particular, if V has dimension h, it does not mix with other operators, and its conformal dimension remains unchanged under the renormalization group flow, then we have

$$\lambda_R \int_{\theta=0}^{2\pi} d\theta V_R(\theta) = \lambda \left(\frac{1}{4}(m+n-2) \right)^{1-h} \int_{\theta=0}^{2\pi} d\theta V(\theta) . \tag{4.22}$$

This semiinfinite cylinder in the u coordinate is nothing but a unit disk D_U with θ labeling the angular parameter along the boundary of the disk, and $e^{-\rho}$ labeling the radial coordinate. Thus what (4.21) represents is the partition function on a unit disk, with the perturbation $\lambda_R \int V_R(\theta) d\theta$ added at the boundary! Notice now that if V is exactly marginal, i.e. if $h = 1$ to all orders, then the perturbation is simply $-\lambda \int_0^{2\pi} d\theta V(\theta)$, whereas if V is a relevant deformation then $h < 1$ and in the

[11] Since this computation involves matter sector fields only, there may be overall constant factors appearing at various stages due to non-zero central charge of the bulk matter theory. These are independent of V and λ and cancel when we compute the ratio of tensions.

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limit $m,n\to\infty$, $\lambda_R\int d\theta V_R(\theta)$ approaches its infrared fixed point $\lambda_{IR}\int d\theta V_{IR}(\theta)$.[12] Thus the net result is that $\langle\Xi^{V,\lambda}_m|\Xi^{V,\lambda}_m\rangle_m$ represents the partition function on the unit disk of the BCFT to which the theory flows in the infrared! As discussed below equation (4.10) this is indeed the tension of the D-brane associated to this BCFT. Thus we conclude that $|\Psi_g\rangle\otimes|\Xi^{V,\lambda}_m\rangle$ gives the D-brane solution corresponding to the BCFT to which the theory would flow in the infrared if we added to the action the boundary perturbation proportional to $\int V(\theta)d\theta$. We emphasize that the string field $|\Psi_g\rangle\otimes|\Xi^{V,\lambda}_m\rangle$ belongs to the state space of BCFT_0.

In particular if we take V to be the identity operator I, we see from eq. (4.13) that

$$\langle\Xi^{I,\lambda}|\phi\rangle=\lim_{n\to\infty}\left\langle\exp\left(-\lambda\int_{\pi/4}^{(\frac{n}{2}-\frac{1}{4})\pi}dx_n\right)f\circ\phi(0)\right\rangle_{\widehat{C}_n}\,,\qquad\forall\,|\phi\rangle\in\mathcal{H}\,. \tag{4.23}$$

For $\lambda>0$, this vanishes due to the infinite range of integration over x_n. Thus we get $|\Xi^{I,\lambda}\rangle=0$ and hence $|\Xi^{I,\lambda}_m\rangle=0$. This is consistent with the fact that the trivial solution $\Psi=0$ represents the tachyon vacuum configuration, and that in the boundary string field theory formalism, perturbation by the identity operator takes the unstable D-brane to the tachyon vacuum.

Note, however, that the solution (4.13) does seem to depend on λ for more general relevant perturbations. Since different values of λ correspond to the same tension of the final brane, we expect that they represent gauge equivalent solutions. Thus the parameter λ is analogous to the redundant parameter b labeling the lower dimensional Dp-brane solutions considered in ref. [30].[13]

4.5 Small deformations of the sliver

From the analysis of subsection 4.3 it is clear that when the operator V is relevant, the state $|\Xi^{V,\lambda}\rangle$ corresponds to a big change in field configuration since it gives rise to a totally new D-brane solution corresponding to the BCFT to which the theory flows in the infrared, whereas if V is exactly marginal, then for small λ, $|\Xi^{V,\lambda}\rangle$ is a solution 'close to' the original solution Ξ.[14] Thus we can define a small deformation $\Delta\Xi^V$ around the solution Ξ through the relation:

$$\Xi^{V,\lambda}=\Xi+\lambda\Delta\Xi^V+O(\lambda^2)\,, \tag{4.24}$$

[12]We shall not consider irrelevant perturbations as it is not clear how to tame the resulting ultraviolet divergences. Since they flow to zero in the IR, such perturbations are not expected to give rise to new solutions.

[13]This was suggested to us by Witten [50].

[14]Even though $\langle\Xi_m|\Xi^{V,\lambda}_m\rangle_m$ vanishes even for small λ, we treat the solutions as 'close' in the sense that the BCFT's corresponding to $\lambda=0$ and λ small have correlation functions which differ from each other by small amount.

and using eqs. (4.13) and (4.14) we find

$$
\begin{aligned}
\langle \Delta\Xi^V|\phi\rangle &= -\lim_{n\to\infty}\left\langle \int_{\frac{\pi}{4}}^{(\frac{n}{2}-\frac{1}{4})\pi} V(x_n)dx_n f\circ\phi(0)\right\rangle_{\widehat{C}_n} \\
&= -\left\langle \int_{|x'|>\frac{\pi}{4}} V(x')dx' f\circ\phi(0)\right\rangle_{D_H}\,, \qquad \forall|\phi\rangle\in\mathcal{H}\,.
\end{aligned}
\tag{4.25}
$$

By expanding eq. (4.15) in powers of λ, it follows that $\Delta\Xi^V$ satisfies

$$
\Xi * \Delta\Xi^V + \Delta\Xi^V * \Xi = \Delta\Xi^V\,. \tag{4.26}
$$

Thus $|\Psi_g\rangle\otimes|\Delta\Xi^V_m\rangle$ will describe small deformation of the string field theory solution describing the D-brane corresponding to BCFT_0. One can also give a direct proof of eq. (4.26) by using the $*$-product rules and the gluing relations (3.35) as follows. For this we shall represent both $\Delta\Xi^V$ through the first of eq. (4.25) and Ξ by a similar formula without the $\int V(x_n)dx_n$ insertion for a fixed finite n and then take $n\to\infty$ limit at the end of the calculation. Using the by now standard procedure we get:

$$
\begin{aligned}
\langle \Delta\Xi^V * \Xi|\phi\rangle &= -\lim_{n\to\infty}\left\langle \int_{\frac{\pi}{4}}^{\pi(\frac{n}{2}-\frac{1}{4})} dx_{2n-1}V(x_{2n-1})\; f\circ\phi(0)\right\rangle_{\widehat{C}_{2n-1}} \\
\langle \Xi * \Delta\Xi^V|\phi\rangle &= -\lim_{n\to\infty}\left\langle \int_{\pi(\frac{n}{2}-\frac{1}{4})}^{\pi(n-\frac{3}{4})} dx_{2n-1}\,V(x_{2n-1})\; f\circ\phi(0)\right\rangle_{\widehat{C}_{2n-1}}\,.
\end{aligned}
\tag{4.27}
$$

Thus we have:

$$
\langle\Delta\Xi^V * \Xi|\phi\rangle + \langle\Xi * \Delta\Xi^V|\phi\rangle = -\lim_{n\to\infty}\left\langle \int_{\frac{\pi}{4}}^{\pi(n-\frac{3}{4})} dx_{2n-1}\,V(x_{2n-1}) f\circ\phi(0)\right\rangle_{\widehat{C}_{2n-1}}\,. \tag{4.28}
$$

Introducing $m=2n-1$ we can rewrite this as

$$
\langle\Delta\Xi^V * \Xi|\phi\rangle + \langle\Xi * \Delta\Xi^V|\phi\rangle = -\lim_{m\to\infty}\left\langle \int_{\frac{\pi}{4}}^{\pi(\frac{m}{2}-\frac{1}{4})} dx_m\,V(x_m)\; f\circ\phi(0)\right\rangle_{\widehat{C}_m}\,. \tag{4.29}
$$

This is precisely the expression (4.25) for $\langle\Delta\Xi^V|\phi\rangle$. Thus we see that $\Delta\Xi^V$ satisfies equation (4.26).

For later use we need to use a regularized expression for $\Delta\Xi^V$ by restricting the integration range in (4.25) to be $|x'|\geq\frac{\pi}{4}+\epsilon$, or equivalently, $\frac{\pi}{4}+\epsilon\leq x_n\leq(\frac{n}{2}-\frac{1}{4})\pi-\epsilon$. When we compute the analog of (4.28) with this regularized expression for $\Delta\Xi^V$, and compare this with the regularized version of (4.25), we find that we miss an integral

$$
-\int_{(\frac{n}{2}-\frac{1}{4})\pi-\epsilon}^{(\frac{n}{2}-\frac{1}{4})\pi+\epsilon} dx_{2n-1}\left\langle V(x_{2n-1}) f\circ\phi(0)\right\rangle_{\widehat{C}_{2n-1}}\,. \tag{4.30}
$$

In the $n\to\infty$ limit the integration region moves off to infinity and the correlator above vanishes as long as V has dimension >0. Thus the regularized $\Delta\Xi^V$ still satisfies eq. (4.26). In the next subsection we shall interpret $\Delta\Xi^V$ as an appropriate covariant derivative of the surface state along a marginal direction in the space of boundary conformal field theories.

4.6 Background independence and theory-space connections

The string field theory action around the tachyon vacuum enjoys a fundamental property: it has manifest background independence with respect to the open string moduli. This observation will allow us to recover the construction of infinitesimal deformations of classical solutions from the point of view of deformations along marginal directions in the open string theory moduli space. To explain these ideas, we need to recall some notions about connections in theory space. Our discussion will follow [51, 55], with some obvious modifications needed for open strings.

Let us fix the CFT in the bulk (the closed string moduli) and for this given bulk CFT, let us consider the space of BCFT's, labeled by some continuous parameters $\{x^s\}$. Each point $\{x^s\}$ specifies a BCFT, with its own state space $\mathcal{H}_x$. We have the structure of a vector bundle, where the base is theory space (parameterized by $\{x^s\}$), and the fiber at each point x is $\mathcal{H}_x$. Thus we can introduce connections on this vector bundle. While there are many possible connections, a natural choice for open string field theory is the canonical connection $\widehat{\Gamma}$ [51]. On surface states, the associated covariant derivative $\widehat{D}_s$ acts as the integration of the exactly marginal perturbation $\mathcal{O}_s$ over the boundary of the Riemann surface, excluding from the integration region the segments around the punctures where local coordinates are defined:

$$\widehat{D}_s \langle\Sigma| = -\int_{\partial\Sigma - \cup_i D_i} dz' \, \langle \Sigma; z' | \mathcal{O}_s \rangle \, . \tag{4.31}$$

Here Σ is a disk with n punctures on its boundary $\partial\Sigma$ with some global coordinate z. The regions $D_i \subset \partial\Sigma$ are the images of the diameters of local coordinates half-disks $\{|\xi_i| \leq 1, \mathrm{Im}(\xi) = 0\}$. The surface state $\langle\Sigma; z'|$ is the $n+1$-punctured disk obtained by introducing an extra puncture on Σ at the boundary point $z = z'$, with local coordinate $\xi = z - z'$. Finally $\mathcal{O}_s$ is an exactly marginal operator of the boundary CFT inserted at this new puncture.

The reflector state[15] ${}_{12}\langle R|$ and the cubic vertex ${}_{123}\langle V_3|$ are covariantly constant with respect to this connection:

$$\widehat{D}_s \, {}_{12}\langle R| = 0 \, , \qquad \widehat{D}_s \, {}_{123}\langle V_3| = 0 \, . \tag{4.32}$$

This follows immediately from the fact that the local coordinate patches cover completely the two and three punctured disks associated to ${}_{12}\langle R|$ and ${}_{123}\langle V_3|$, so that in both cases $\partial\Sigma - \cup_i D_i$ is the empty set. We can rephrase (4.32) by saying that $\widehat{D}_s$ acts as a derivation of the $*$ product:

$$\widehat{D}_s(\phi_1 * \phi_2) = \widehat{D}_s\phi_1 * \phi_2 + \phi_1 * \widehat{D}_s\phi_2 \, , \tag{4.33}$$

where ϕ_1 and ϕ_2 are two sections of the vector bundle.

[15] The reflector ${}_{12}\langle R|$ is the two-punctured disk with punctures at $z = 0$ and $z = \infty$ in UHP coordinates, with local coordinates $\xi_1 = z$ around $z = 0$ and $\xi_2 = -1/z$ around $z = \infty$. The reflector provides the definition of the BPZ inner product: $\langle\psi|\phi\rangle \equiv {}_{12}\langle R| \, \psi\rangle_1 \, |\phi\rangle_2$.

At each point x in theory space, exactly marginal deformations are represented by dimension one primaries $\mathcal{O}_s$ of the *matter* BCFT. Thus for each exactly marginal direction s, the covariant derivative $\widehat{D}_s$ does not involve any ghosts and commutes with the purely ghost kinetic operator $\mathcal{Q}$ of vacuum string field theory,

$$[\widehat{D}_s, \mathcal{Q}] = 0\,. \tag{4.34}$$

The string field action (2.1) is naturally a function on the vector bundle. $\mathcal{S}(\Psi)$ is indeed $\mathcal{S}(\psi^i, x)$ where ψ^i are components along the vectors in $\mathcal{H}_x$. It can be written as

$$\mathcal{S}(\Psi, x) \equiv -\frac{1}{g_0^2}\left[\frac{1}{2}\langle R|\Psi\rangle \mathcal{Q}|\Psi\rangle + \frac{1}{3}\langle V_3|\Psi\rangle|\Psi\rangle|\Psi\rangle\right]. \tag{4.35}$$

It follows from the definition of a covariant derivative of functions on a vector bundle (such derivative measures how the function changes as we move on the base, and on the fiber by parallel transport) and relations (4.32) and (4.34) that

$$\widehat{D}_s\,\mathcal{S} = 0\,. \tag{4.36}$$

This is the statement of manifest background independence of the vacuum string field theory: there is a (canonical) connection $\widehat{\Gamma}$ so that the action is covariantly constant. This means that vacuum string field theory is independent of the choice of the reference boundary conformal field theory BCFT_0 — string field actions using nearby BCFT′s are identical when the string fields are related by an homogeneous field redefinition generated by parallel transport with the connection $\widehat{\Gamma}$. This is simpler than for conventional string field theory [55], where the covariant derivative of the action is nonvanishing and the requisite field redefinition includes a constant shift and other terms in addition to parallel transport.

Consider now a solution $\Psi^x = \Psi_g \otimes \Psi^x_m$ of the vacuum string field theory equations corresponding to a certain BCFT background labeled by x, *expressed as an element of* $\mathcal{H}_x$. $|\Psi^x_m\rangle$ is simply the matter part of the sliver state $|\Xi^x_m\rangle$ associated with this conformal field theory. Thus we have

$$\mathcal{Q}\Psi^x + \Psi^x * \Psi^x = 0\,, \qquad \Psi^x_m *^m \Psi^x_m = \Psi^x_m\,. \tag{4.37}$$

Since Ψ^x is defined as an element of $\mathcal{H}_x$ for every x, we have a section of the vector bundle introduced earlier. Using the fact that $\widehat{D}_s|\Psi_g\rangle = 0$, we have

$$\widehat{D}_s|\Psi^x\rangle = |\Psi_g\rangle \otimes \widehat{D}_s|\Xi^x_m\rangle\,. \tag{4.38}$$

Since $|\Xi^x_m\rangle$ is a surface state, its covariant derivative is given by eq. (4.31). Comparing this with (4.25) we see that this is just $|\Delta\Psi^{\mathcal{O}_s}\rangle$. Thus we have

$$\widehat{D}_s|\Psi^x\rangle = |\Psi_g\rangle \otimes |\Delta\Psi^{\mathcal{O}_s}\rangle\,. \tag{4.39}$$

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The small deformation of the sliver solution considered in the previous subsection is simply the covariant derivative of Ψ^x with the connection $\widehat{\Gamma}$. Indeed, applying $\widehat{D}_s$ on the second of eq. (4.37), and using eqs. (4.33), one gets

$$(\widehat{D}_s\Psi^x_m) *^m \Psi^x_m + \Psi^x_m *^m (\widehat{D}_s\Psi^x_m) = \widehat{D}_s\Psi^x_m\,. \tag{4.40}$$

This shows that $\widehat{D}_s\Psi^x$ satisfies the small fluctuation equation (4.26).

Although we have been careful in phrasing the discussion in terms of *exactly* marginal deformations, as only for those it makes sense to talk about connections in theory space, for *infinitesimal* deformations we can nevertheless consider the generalization that $\mathcal{O}_s$ is a generic dimension one matter primary. The operator $\widehat{D}_s$ is still well-defined and obeys (4.33) and (4.34). So we obtain a solution $\widehat{D}_s\Psi^x$ of the small fluctuation equations for each dimension one matter primary. These states are promising candidates for the spectrum of physical excitations around the D-brane background. We shall discuss this in detail in section 5.

It is intriguing that some of the previous analysis appears to apply formally to closed string deformations. A closed string deformation of a surface state is obtained by integrating a closed string marginal operator $\mathcal{O}_{\bar{s}}$ over the *bulk* of the surface state, leaving out the local coordinate regions. It would seem that such deformation of the sliver would satisfy the small fluctuation equation. In this case, however, this integral is plagued with ultraviolet divergences from the region of integration where the operator $\mathcal{O}_{\bar{s}}$ comes close to the boundary of the disk, and it is not *a priori* clear how such divergences should be regulated. This deformation — if it can be defined — might just be the open string by-product of a shift in the closed string background, as is possibly the case in open/closed string field theory [62]. But perhaps it represents some true encoding of closed string physics into the open string state space. It is worth noting that the sliver, as opposed to the identity string field used earlier for encoding closed string deformations in open string theory [63], has plenty of bulk to insert closed string states. Clearly this issue deserves further investigation.

5. Physical states around D-brane backgrounds

Given a candidate solution representing a specific D-brane, one would like to find the spectrum of physical states around the D-brane background and compare this with the known spectrum of physical open string states in that particular D-brane background. Let $\Psi_0 = \Psi_g \otimes \Psi_m$ be a solution representing a particular D-brane in the string field theory around the tachyon vacuum [30]. After shifting the string field $\Psi = \Psi_0 + \Phi$, the action takes the form:

$$\mathcal{S}(\Phi) \equiv -\frac{1}{g_0^2}\left[\frac{1}{2}\langle\Phi, \mathcal{Q}_0\,\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right], \tag{5.1}$$

where the action of the BRST operator $\mathcal{Q}_0$ in the new background on any state $|A\rangle$ in $\mathcal{H}$ is given as

$$\mathcal{Q}_0\, A = \mathcal{Q}A + \Psi_0 * A - (-)^A A * \Psi_0\,. \tag{5.2}$$

Around the new background the physical state condition, corresponding to the linearized equations of motion, is then

$$\mathcal{Q}_0\Phi = \mathcal{Q}\Phi + \Psi_0 * \Phi + \Phi * \Psi_0 = 0\,, \tag{5.3}$$

while the linearized gauge symmetries are given as

$$\delta_\Lambda \Phi = \mathcal{Q}_0\Lambda = \mathcal{Q}\Lambda + \Psi_0 * \Lambda - \Lambda * \Psi_0\,. \tag{5.4}$$

Physical states around Ψ_0 are identified with the cohomology classes of $\mathcal{Q}_0$.

Clearly a full solution of this problem requires the knowledge of $\mathcal{Q}$, and the solution Ψ_0 including its ghost part, which are not available to us at present. In this section we shall try to get some insight into this problem.

5.1 Factorization ansatz for fluctuations

Clearly there are two parts to the problem. First we need to find solutions to the linearized equations of motion (5.3) and then we need to determine which of these are related by gauge transformations. For the first problem, we could look at a subset of field configurations of the form:

$$\Phi = \Psi_g \otimes \delta\Psi_m\,, \tag{5.5}$$

where $\delta\Psi_m$ is a matter state. Replacing (5.5) into (5.3) and using eq. (2.6) we find the condition

$$\Psi_m *^m \delta\Psi_m + \delta\Psi_m *^m \Psi_m = \delta\Psi_m\,. \tag{5.6}$$

Note that this condition does not involve the operator $\mathcal{Q}$. Clearly not every solution of (5.3) is of the form (5.5). However it is conceivable that every solution of (5.3) can be brought to this form using a gauge transformation (5.4), i.e. there is a representative element of the form (5.5) in every cohomology class of $\mathcal{Q}_0$.

It is possible to find solutions to eq. (5.6). For definiteness we shall take Ψ_m to be Ξ_m, — the matter part of the sliver state associated with BCFT$_0$, — and suppose that it represents a particular D-brane in a moduli space that includes some marginal directions. As discussed in section 4, the various points in this moduli space can be obtained as deformed sliver solutions. Defining $\Delta\Xi_m^V$ as the matter part of $\Delta\Xi^V$ defined in section 4.5 and using eq. (4.26) we see that $\delta\Psi_m \propto \Delta\Xi_m^V$ satisfies eq. (5.6). Although the argument given above uses an exactly marginal deformation, the explicit analysis of section 4.5, showing that $\Delta\Xi^V$ satisfied eq. (4.26), goes through if we choose V to be an arbitrary dimension one primary operator. As a result $\delta\Psi_m \propto \Delta\Xi_m^V$ still satisfies eq. (5.6). Thus we seem to have gotten a solution of the linearized equations of motion for every dimension one matter primary operator in BCFT$_0$.

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While this is encouraging, there are some subtleties which we now discuss. First of all, we compute the inner product $\langle \Delta\Xi^V_m | \Delta\Xi^V_m \rangle$ as in eq. (4.20). This is given by:

$$\langle \Delta\Xi^V_m | \Delta\Xi^V_m \rangle = \lim_{m,n\to\infty} \Big\langle \Big(\int_{\frac{\pi}{4}}^{(\frac{m}{2}-\frac{1}{4})\pi} V(x_{m+n-2}) dx_{m+n-2} \Big) \times \\ \times \Big(\int_{(\frac{m}{2}-\frac{1}{4})\pi}^{\frac{m+n-1}{2}\pi - \frac{\pi}{4}} V(x_{m+n-2}) dx_{m+n-2} \Big) \Big\rangle_{\widetilde{C}_{m+n-2}} . \tag{5.7}$$

The correlation functions are computed in the matter sector only. In the $m, n \to \infty$ limit the range of integration over x_{m+n-2} is infinite, but this is easily avoided by going to a new coordinate system $u \equiv i\rho + \theta = 4(z_{m+n-2} - \pi/4)/(m+n-2)$ which maps $\widetilde{C}_{m+n-2}$ to a seminfinite cylinder C of circumference 2π. Dimension one primary fields will not pick up any factor under this rescaling, and we get

$$\langle \Delta\Xi^V_m | \Delta\Xi^V_m \rangle = \Big\langle \Big(\int_0^{\pi} V(\theta) d\theta \Big) \Big(\int_{\pi}^{2\pi} V(\theta') d\theta' \Big) \Big\rangle_C . \tag{5.8}$$

Note now that the range of integration over θ and θ' coincide at two points, $\theta = 0$ and $\theta = \pi$. Since $\langle V(\theta) V(\theta') \rangle \sim |\theta - \theta'|^{-2}$ for small $|\theta - \theta'|$, this gives logarithmically divergent integrals. Thus the BPZ norm of $\Delta\Xi^V_m$ in the matter sector is logarithmically divergent. This of course is related to the ultraviolet divergence discussed in section 4 and one could try to regularize this by taking the limit of x' integration in eq. (4.25) to be in the range $|x'| \geq \pi/4 + \epsilon$. As discussed in section 4.5, $\Delta\Xi^V$ satisfies eq. (4.26) even when ϵ is finite. However unlike $\Xi^{V,\lambda}_m$, which has finite norm for finite ϵ (when measured with respect to the norm of Ξ_m), $\Delta\Xi^V_m$ has infinite norm even for finite ϵ. To see this we note that the rescaling needed to go from the x_{m+n-2} coordinate to the θ coordinate shrinks a region of finite width to zero width. More precisely, with the regulator ϵ in place and after the rescaling, we get

$$\langle \Delta\Xi^V_m | \Delta\Xi^V_m \rangle = \lim_{m,n\to\infty} \Big\langle \Big(\int_{4\epsilon/(m+n-2)}^{\pi - 4\epsilon/(m+n-2)} V(\theta) d\theta \Big) \Big(\int_{\pi + 4\epsilon/(m+n-2)}^{2\pi - 4\epsilon/(m+n-2)} V(\theta') d\theta' \Big) \Big\rangle_C . \tag{5.9}$$

Thus even for finite ϵ the integral has logarithmic divergences in the $m, n \to \infty$ limit, since the integration limits approach each other. This divergence is actually needed for consistency. Indeed, note that for a marginal deformation $\Xi^{V,\lambda}_m$ can be identified with Ξ'_m associated with a deformed BCFT$'$ discussed in section 4.1. The norm of Ξ'_m, being equal to $(2\epsilon)^{-4h}$ times the partition function of BCFT$'$ on the cylinder, approaches that of Ξ_m for small λ since as $\lambda \to 0$, $h \to 0$ and the partition functions of BCFT$'$ and BCFT$_0$ are equal. The orthonormality of Ξ_m and $\Xi^{V,\lambda}_m = \Xi'_m$ (section (4.1)) then leads to the result that the norm of $(\Xi^{V,\lambda}_m - \Xi_m)$ is twice the norm of Ξ_m in the $\lambda \to 0$ limit. Given that $\Xi^{V,\lambda}_m = \Xi_m + \lambda \Delta\Xi^V_m + \mathcal{O}(\lambda^2)$, we reach the conclusion that λ^2 times the norm of $\Delta\Xi^V_m$ is twice the norm of Ξ_m in the small λ limit. If this has to be true for arbitrarily small λ, then $\Delta\Xi^V_m$ must have infinite norm.

This divergence is not necessarily problematic, since it is not clear *a priori* that having finite norm in the matter sector is a requirement on physical deformations. As we have seen in section (4.1), the deformed solution Ξ'_m does have finite norm, even if $\Delta\Xi^V_m$ has divergent norm. We shall also argue later that having a logarithmically divergent norm of this kind may be a necessary condition for a physical deformation not to be pure gauge. From a physical viewpoint, since we have seen that for exactly marginal V, $\Delta\Xi^V$ represents some small deformation of BCFT_0, we are led to believe that such deformations must be allowed in the computation of the spectrum.

The second subtlety arises because the manipulations in section 4.5 leading to eq. (4.26) actually hold without any constraint on V as the gluing in the $\widehat{z}_n$ coordinate system does not require us to make any conformal transformations at all. So one might ask: why can't we use the matter part of these states to find new solutions of eq. (5.6)? The answer to this question may again be hidden in the normalization of the state. For this, let us write down the norm of $\Delta\Xi^V_m$ for an operator V of arbitrary dimension h:

$$\begin{aligned}\langle\Delta\Xi^V_m|\Delta\Xi^V_m\rangle = \lim_{m,n\to\infty}\Bigg[\Big(\frac{m+n-2}{4}\Big)^{2(1-h)}\Bigg\langle\Big(\int_{4\epsilon/(m+n-2)}^{\pi-4\epsilon/(m+n-2)} V(\theta)d\theta\Big)\times \\ \times\Big(\int_{\pi+4\epsilon/(m+n-2)}^{2\pi-4\epsilon/(m+n-2)} V(\theta')d\theta'\Big)\Bigg\rangle_C\Bigg].\end{aligned} \tag{5.10}$$

Note the prefactor coming from the conformal transformation of the vertex operators when we make the final conversion from x_{m+n-2} to θ coordinate. For $h<1$ there is no short distance divergence from the limits of integration of θ, θ', but the contribution from the finite θ region has power law divergence due to the infinite prefactor in the $m,n\to\infty$ limit. Thus the divergence in the norm of such states is worse than that of a marginal deformation, and clearly originates from the large distance region (in the original x_{m+n-2} coordinate system.) In fact this divergence is precisely the reason why a small perturbation by V drives the system all the way to the infrared fixed point. Thus we can conclude with reasonable degree of confidence that for $h<1$ the $\Delta\Xi^V$ does not represent small deformation. On the other hand for $h>1$ the integral is finite for finite ϵ as the short distance singularity coming from the region of integration $\theta\simeq\theta'$ at 0 or π cancel the conformal factor exactly. Thus $\Delta\Xi^V$ has finite norm. This seems to make irrelevant perturbations better behaved. As we shall see below, however, these are most likely pure gauge deformations, precisely because they have finite norm.

5.2 Factorization ansatz for gauge transformations

Not all solutions of (5.6) represent inequivalent physical states, since we still need to consider the linearized gauge invariances (5.4). A gauge transformation (5.4) with generic parameter Λ will not respect the form (5.5) of the fluctuations. While there may be more general gauge parameters respecting (5.5) we now restrict ourselves to factored gauge parameters $\Lambda = \Lambda_g \otimes \Lambda_m$, where Λ_m is a matter state and Λ_g is a ghost state satisfying

$$\mathcal{Q}\Lambda_g = 0\,, \qquad \Lambda_g *^g \Psi_g = \Psi_g *^g \Lambda_g = \Psi_g\,. \tag{5.11}$$

For such a Λ_g, (5.4) gives

$$\begin{aligned} \delta_\Lambda(\Phi) &= \mathcal{Q}\Lambda_g \otimes \Lambda_m + (\Psi_g *^g \Lambda_g) \otimes (\Xi_m *^m \Lambda_m - \Lambda_m *^m \Xi_m) \\ \to \quad \Psi_g \otimes \delta_\Lambda \delta\Psi_m &= \Psi_g \otimes (\Xi_m *^m \Lambda_m - \Lambda_m *^m \Xi_m)\,. \end{aligned} \tag{5.12}$$

It follows from this equation and from (5.6) that we can study physical states around D-brane backgrounds by considering the space of equivalence classes $\mathcal{H}_{\delta\Psi}$ of matter string fields $\delta\Psi_m$ which satisfy the equations:

$$\Xi_m *^m \delta\Psi_m + \delta\Psi_m *^m \Xi_m = \delta\Psi_m \tag{5.13}$$

$$\delta\Psi_m \simeq \delta\Psi_m + \Xi_m *^m \Lambda_m - \Lambda_m *^m \Xi_m\,. \tag{5.14}$$

We should keep in mind, however, that there may be gauge transformation parameters other than those of the form $\Lambda_g \otimes \Lambda_m$ which generate factorized deformations of the form $\Psi_g \otimes \delta\Psi_m$. These will generate further equivalence relations between the deformations $\delta\Psi_m$.

Does a Λ_g satisfying (5.11) exist? Let $\mathcal{I}_g$ denote the ghost part of the identity string state, normalized so that $\mathcal{I}_g *^g \mathcal{I}_g = \mathcal{I}_g$. If $\mathcal{Q}$ annihilates the identity state, then $\Lambda_g = \mathcal{I}_g$ clearly satisfies eq. (5.11). We shall now argue that $\Lambda_g = \mathcal{I}_g$ is also the most natural choice. From the analysis of refs. [56]–[61][31, 49] we know that barring the complications of the string mid-point coordinate and/or zero modes, it is natural to think of the string field as a matrix acting on the half string state space. In particular we can think of the factored string field of the form (2.5) as the direct product of two matrices, one acting on the ghost sector of the half-string state and the other acting on the matter sector of the half-string state space. Since the identity string field $\mathcal{I}$ can be regarded as an identity matrix on the half string state space, choosing $\Lambda = \mathcal{I}_g \otimes \Lambda_m$ corresponds to a matrix which is a direct product of the identity matrix in the ghost sector of the half-string state space and generator of a non-trivial rotation in the matter sector of the half-string state space. This is precisely how we would represent the rotation generator on a product of two vector spaces if we want to construct generators that act on only one of the two vector spaces. Thus from this physical point of view choosing $\Lambda_g = \mathcal{I}_g$ would be most natural. This in turn, would require us to choose $\mathcal{Q}$ that annihilates $\mathcal{I}_g$. We take this as a strong argument in favor of choosing such $\mathcal{Q}$.[16] Given this choice we can now proceed by taking (5.14) to be a valid gauge equivalence relation between different $\delta\Psi_m$.

[16]The result of ref. [24], showing that around the tachyon vacuum $\mathcal{I}$ is a pure gauge state, also requires us to choose such a $\mathcal{Q}$.

Since these equations refer to matter fluctuations $\delta\Psi_m$, without risk of confusion we shall from now on drop all m subscripts and superscripts referring to matter and write these equations as

$$\Xi * \delta\Psi + \delta\Psi * \Xi = \delta\Psi \tag{5.15}$$

$$\delta\Psi \simeq \delta\Psi + \Xi * \Lambda - \Lambda * \Xi \,. \tag{5.16}$$

In the next two subsections we discuss some consequences of these equations. All states, $*$-products and correlation functions in these subsections will refer to matter sector only.

5.3 Deformed projectors and rules for physical states

The physical state conditions (5.15) obtained above have the interpretation of deformation equations for projectors, with a familiar equivalence relation. Indeed this equation is clearly equivalent to the condition that $(\Xi + \delta\Psi)$ be a projector:

$$(\Xi + \delta\Psi) * (\Xi + \delta\Psi) = (\Xi + \delta\Psi) + \mathcal{O}(\delta\Psi^2)\,. \tag{5.17}$$

Similarly, equation (5.16) is equivalent to

$$\Xi + \delta\Psi \simeq e^{-\Lambda} * \Xi * e^{\Lambda} + \mathcal{O}(\Lambda^2)\,, \tag{5.18}$$

where in the expansion of e^{Λ} products correspond to $*$-products. Therefore our physical state conditions are conditions for deformed projectors with a $U(\infty)$ local gauge symmetry.

This leads to a puzzle. It is simple to show formally that the above physical state problem has no nontrivial solutions! Indeed, we multiply (5.15) from the left by Ξ and use associativity and the projector condition for Ξ to obtain

$$\Xi * \delta\Psi * \Xi = 0\,. \tag{5.19}$$

It then suffices to take

$$\Lambda = \Xi * \delta\Psi - \delta\Psi * \Xi\,, \tag{5.20}$$

to verify that

$$\begin{aligned}\Xi * \Lambda - \Lambda * \Xi &= \Xi * \delta\Psi - 2\,\Xi * \delta\Psi * \Xi + \delta\Psi * \Xi \\ &= \Xi * \delta\Psi + \delta\Psi * \Xi = \delta\Psi\,,\end{aligned} \tag{5.21}$$

where we used associativity, equation (5.19) and (5.15). It follows that $\delta\Psi$ is trivial.

This clearly shows that if eq. (5.16) is the correct equivalence relation, we cannot get non-trivial elements of the cohomology using states which are completely regular. One way to avoid this problem is to use a non-normalizable $\delta\Psi$. This will typically give rise to non-normalizable Λ of the same type via eq. (5.20). If the requirement of normalizability on string field fluctuations is different from that on the gauge transformation parameters (which is not improbable considering that the two are multiplied by different ghost sector states) it is possible to have a non-normalizable $\delta\Psi$ which is an allowed deformation, while the corresponding Λ constructed through eq. (5.20) is not an allowed gauge transformation parameter.

Indeed, this suggestion is physically motivated. We have argued earlier that despite having logarithmically divergent norms, deformations $\delta\Psi \propto \Delta\Xi^V_m$ associated with marginal operators V should be considered as physical deformations. It is easy to verify that Λ constructed from such a $\delta\Psi$ through eq. (5.20) also has logarithmically divergent norm. If such non-normalizable Λ's did generate gauge transformations, then we would be forced to conclude that two different BCFT's related by a marginal deformation are gauge equivalent. Since this is not physically acceptable, we are led to the conclusion that matter sector states Λ with logarithmically divergent norms are not valid gauge transformation parameters. This principle will also explain why the state associated with deformations by irrelevant operators, found at the end of section 5.1, are pure gauge; they have finite norm and the corresponding gauge transformation parameter Λ defined through eq. (5.20) also has finite norm. Thus such deformations can be gauged away.

While this principle leads us to identify deformations of the sliver by dimension-one primaries as physical states, it also includes in the list deformations by unwanted dimension-one operators — the vertex operators of null states and non-primary states. At present it is not clear how to eliminate them and reproduce the correct spectrum of physical states on the D-brane.[17]

6. Discussion

In this concluding section we make some general remarks about the string field theory action given in eq. (2.1) and also discuss the implication of the results of the present paper in a general context. We end with a discussion of some of the open problems in this subject.

- Typically the formulation of cubic open string field theory requires a choice of background, determining the form of the quadratic term in the action. In that sense the action (2.1) represents the choice of the tachyon vacuum as the background around which we expand. But this clearly is a special background being the end-point of tachyon condensation of any D-brane. The action is formally independent of the choice of BCFT whose basis we use to expand the string field, as is apparent from the fact that $\mathcal{Q}$ is made purely of ghost operators, and the $*$-product, defined through overlap conditions on string wave-functionals,

[17]Although a null state has vanishing two point function with itself, its two point function with a vertex operator which is neither a primary nor a secondary, could be non-zero.

is formally independent of the choice of open string background. At least for backgrounds related by exactly marginal deformations, this notion of manifest background independence can be made precise using the language of connections in theory space [51, 55], as we demonstrated in section 4.6. Starting from the tachyon vacuum we can reach any D-brane configuration by simply considering the sliver associated with that particular BCFT. Note, however, that the spirit here is somewhat different from that of boundary string field theory, — in our approach we always represent the string field as an element of the state space of a specific reference BCFT. At the end it should not matter which basis we use to expand the string field. Indeed, we have seen in section 4 that physical quantities like ratios of tensions of D-branes do not depend on this choice.

- The structure of the string field theory action (2.1) is very similar in spirit to the action of p-adic string theory [64, 65, 66]. Both are non-local, and in both cases the action expanded around the tachyon vacuum is perfectly non-singular and has no physical excitations. Yet in both cases the theory admits lump solutions which support open string excitations. The Dp-brane solutions are gaussian in the case of p-adic string theory, and also in the case of vacuum string field theory, although in this case the string field has many components corresponding to different oscillator excitations. It will be interesting to explore if there is any deeper significance to this similarity.

- Vacuum string field theory is much simpler than conventional cubic open string field theory. Explicit analytic solutions of equations of motion are possible, as we saw in this and the earlier papers. Also in this theory off-shell 'tachyon' amplitudes (and perhaps other amplitudes as well) around the tachyon vacuum can be computed exactly up to overall normalization. All this is commensurate with the fact that we have chosen to expand the action around a simpler background. In this context we would like to note that even the p-adic string action takes a simple form only when expended about the tachyon vacuum. Once we introduce shifted fields to expand the action around the D-brane background, the action looks much more complicated.

- Clearly the most pressing problem at this stage is understanding the ghost sector. This is needed not merely to complete the construction of the action, but also for understanding gauge transformations in this theory. This, in turn is needed for classifying inequivalent classical solutions and the spectrum of physical states around D-brane backgrounds. The knowledge of $\mathcal{Q}$ will also enable us to calculate quantum effects in this theory and determine whether the theory contains in its full spectrum, the elusive closed string states. On this issue, at the end of section 4.6 we speculated whether integration of dimension $(1,1)$

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closed string primaries over the bulk of the sliver could yield a representation of closed string degrees of freedom in the open string state space.

In this paper we have seen that vacuum SFT incorporates nicely the most attractive features of boundary SFT — the automatic generation of correct tensions, and the description of solutions in terms of renormalization group ideas. These features arise in vacuum string field theory by taking into account the unusual geometrical definition of the sliver state. The two present shortcomings of boundary SFT — the tachyon vacuum being a singular endpoint of the configuration space, and the difficulty of defining the theory in the space of all backgrounds — are avoided in vacuum SFT. In a previous paper [31], we noted the remarkable simplicity of the algebraic approach to the construction of D-brane configurations in vacuum SFT. All in all, we are led to believe that vacuum string field theory may provide a surprisingly powerful and flexible approach to non-perturbative string theory.

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